Note:

CHAPTER 2: DIMENSIONLESS

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This chapter is part of the textbook:

"Fundamentals of Die Casting Design"

You can download the whole book if you like from: *www.potto.org.*

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APRIL 1, 2009

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Dynamics	NSY		0.0.0	✖	-
Fluid Mechanics	alpha		0.1.8	✔	15,000
Heat Transfer	NSY	Based on Eckert	0.0.0	✖	-
Mechanics	NSY		0.0.0	✖	-
Open Channel Flow	NSY		0.0.0	✖	-
Statics	early alpha	first chapter	0.0.1	✖	-
Strength of Material	NSY		0.0.0	✖	-
Thermodynamics	early alpha		0.0.01	✖	-
Two/Multi phases flow	NSY	Tel-Aviv'notes	0.0.0	✖	-

NSY = Not Started Yet

"The shear, S, at the ingate is determined by the average velocity, U, of the liquid and by the ingate thickness, t. Dimensional analysis shows that is directly proportional to (U/ℓ). The constant of proportionality is difficult to determine, . . .[1]"

Murray, CSIRO Australia

CHAPTER 3

Dimensional Analysis

One of the important tools to understand the die casting process is dimensional analysis. Fifty years ago, this method transformed the fluid mechanics/heat transfer into a "uniform" understanding. This book attempts to introduce to the die casting industry this established method[2]. Experimental studies will be "expanded/generalized" as it was done in convective heat transfer. It is hoped that as a result, separate sections for aluminum, zinc, and magnesium will not exist anymore in die casting conferences. This chapter is based partially on Dr. Eckert's book, notes, and the article on dimensional analysis applied to die casting. Several conclusions are derived from this analysis and they will be presented throughout this chapter. This material can bring great benefit to researchers who want to built their research on a solid foundation. For those who are dealing with the numerical research/calculation, it is useful to learn when some parameters should be taken into account and why.

[1] Citing "The Design of feed systems for thin walled zinc high pressure die castings," Metallurgical and materials transactions B Vol. 27B, February 1996, pp. 115–118. This excerpt is an excellent example of poor research and poor understanding. This "unknown" constant is called viscosity (see Basics of Fluid Mechanics in Potto series. Here, a discussion on some specific mistakes were presented in that paper (which are numerous). Dimensional analysis is a tool which can take "cluttered" and meaningless paper such as the above and turn them into something with real value. As proof of their model, the researchers have mentioned two unknown companies that their model is working. What a nice proof! Are the physics laws really different in Australia?

[2] Actually, Prof. E.R.G. Eckert introduced the dimensional analysis to the die casting long before. The author is his zealous disciple, all the credit should go to Eckert. Of course, all the mistakes are the author's and none of Dr. Eckert's. All the typos in Eckert's paper were this author's responsibility for which he apologizes.

3.0.7 How The Dimensional Analysis Work

In dimensional analysis, the number of the effecting parameters is reduced to a minimum by replacing the dimensional parameters by dimensionless parameters. Some researchers point out that the chief advantage of this analysis is "to obtain experimental results with a minimum amount of labor, results in a form having maximum utility" [18, pp. 395]. The dimensional analysis has several other advantages which include; 1)increase of understanding, 2) knowing what is important, and 3) compacting the presentation[3]. The advantage of compact of presentation allows one to "see" the big picture with minimal effort.

Dimensionless parameters are parameters which represent a ratio which does not have a physical dimension. The experimental study assists to solve problems when the solution of the governing equation cannot be obtained. To achieve this, experiments are designed to be "similar" to the situations which need to be solved or simulated. The base for this concept is mathematical. Two different sets of phenomena will produce a similar result if the governing differential equations with boundaries conditions are similar. The actual experiments are difficult to carry out in many cases. Thus, design experiments with the same governing differential equations as the actual phenomenon is the solution. This similarity does not necessarily mean that the experiments have to be carried exactly as studied phenomena. It is enough that the main dimensionless parameters are similar, since the minor dimensional parameters, in many cases, are insignificant. For example, a change in Reynolds number is insignificant since a change in Reynolds number in a large range does not affect the friction factor.

An example of the similarity applied to the die cavity is given in the section 3.5. Researchers in casting in general and die casting in particular do not utilize this method. For example, after the Russians [6] introduced the water analogy method (in casting) in the 40s all the experiments such as Wallace, CSIRO, etc. conducted poorly designed experiments. For example, Wallace record the Reynolds and Froude number without attempting to match the governing equations. Another example is the experimental study of Gravity Tiled Die Casting (low pressure die casting) performed by Nguyen's group in 1986 comparing two parameters Re and We. Flow of "free" falling, the velocity is a function of the height ($U \sim \sqrt{gH}$). Hence, the equation $Re_{model} = Re_{actual}$ should lead only to $H_{model} \equiv H_{actual}$ and not to any function of U_{model}/U_{actual}. The value of U_{model}/U_{actual} is actually constant for the same height ratio. The Wallace experiments with Reynolds number matching does not lead to matching of similar governing equations. Many other important parameters which control the governing equations are not simulated [26]. The governing equations in these cases include several other important parameters which have not been controlled

[3]The importance of compact presentation is attributed to Prof. M. Bentwitch who was mentor to many including the author during his masters studies.

or even measured, monitored, and simulated[4]. Moreover, the Re number is controlled by the flow rate and the characteristics of the ladle opening and not as in the pressurized pipe flow as the authors assumed.

3.1 Introduction

Lets take a trivial example of fitting a rode into a circular hole (see Figure 3.1). To solve this problem, it is required to know two parameters; 1) the diameter of the rode and 2) the diameter of the hole. Actually, it is required to have only one parameter, the ratio of the rode diameter to the hole diameter. The ratio is a dimensionless number and with this number one can say that for a ratio larger than one, the rode will not enter the hole; and ratio smaller than one, the rod is too small. Only when the ratio is equal to one, the rode is said to be fit. This allows one to draw the situation by using only one coordinate. Furthermore, if one wants to deal with tolerances, the dimensional analysis can easily be extended to say that when the ratio is equal from 0.99 to 1.0 the rode is fitting, and etc. If one were to use the two diameters description, he will need more than this simple sentence to describe it.

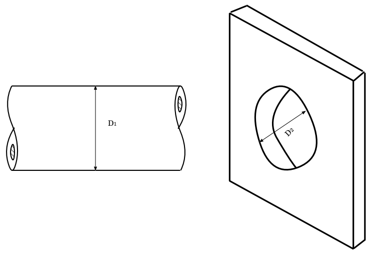

Fig. -3.1. Rod into the hole example

In the preceding simplistic example, the advantages are minimal. In many real problems, including the die casting process, this approach can remove clattered views and put the problem into focus. It also helps to use information from different problems to a “similar” situation. Throughout this book the reader will notice that the systems/equations are converted to a dimensionless form to augment understanding.

3.2 The Die Casting Process Stages

The die casting process can be broken into many separated processes which are controlled by different parameters. The simplest division of the process for a cold chamber is the following: 1) filling the shot sleeve, 2) slow plunger velocity, 3) filling the runner system 4) filling the cavity and overflows, and 5) solidification process (also referred as intensification process). This division into such sub–processes results in a clear picture on each process. On one hand, in processes 1 to 3, it is desirable to have a minimum

[4]Besides many conceptual physical mistakes, the authors have a conceptual mathematical mistake. They tried to achieve the same Re and Fr numbers in the experiments as in reality for low pressure die casting. They derived an equation for the velocity ratio based on equal Re numbers (model and actual). They have done the same for Fr numbers. Then they equate the velocity ratio based on equal Re to velocity ratio based on equal Fr numbers. However, velocity ratio based on equal Re is a constant and does vary with the tunnel dimension (as opposed to distance from the starting point). The fact that these ratios have the same symbols do not mean that they are really the same. These two ratios are different and cannot be equated.

heat transfer/solidification to take place for obvious reasons. On the other hand, in the rest of the processes, the solidification is the major concern.

In die casting, the information and conditions do not travel upstream. For example, the turbulence does not travel from some point at the cavity to the runner and of–course, to the shot sleeve. This kind of relationship is customarily denoted as a parabolic process (because in mathematics the differential equations describe these kind of cases as parabolic). To a larger extent it is true in die casting. The pressure in the cavity does not affect the flow in the sleeve or the runner if the vent system is well designed. In other words, the design of the pQ^2 diagram is not controlled by down–stream conditions. Another example, the critical slow plunger velocity is not affected by the air/gas flow/pressure in the cavity. In general, the turbulence generated down–stream does not travel up–stream in this process. One has to restrict this characterization to some points. One point is particularly mentioned here: The poor design of the vent system affects the pressure in the cavity and therefore the effects do travel down stream. For example, the pQ^2 diagram calculations are affected by poor vent system design.

3.2.1 Filling the Shot Sleeve

The flow from the ladle to the shot sleeve did not receive much attention in the die casting research[5] because it is believed that it does not play a significant role. For low pressure die casting, the flow of liquid metal from the ladle through "channel(s)" to the die cavity plays an important role[6]. The importance of the understanding of this process can show us how to minimize the heat transfer, layer created on the sleeve (solidification layer), and sleeve protection from; a) erosion b) plunger problem. The jet itself has no smooth surface and two kinds of instability occurs. The first instability is of Bernoulli's effect and second effect is Bar-Meir's effect that boundary conditions cannot be satisfied for two phase flow. Yet, for die casting process, these two effects (see Figure 3.2 do not change the global flow in the sleeve. At first, the hydraulic jump is created when the liquid metal enters the sleeve. The typical time scale for hydraulic jump creation is almost instant and extremely short as can be shown by the characteristic methods. As the liquid metal level in the sleeve rises, the location of the jump moves closer to the impinging center. At a certain point, the liquid depth level is over the critical depth level and the hydraulic jump disappears. The critical depends on the liquid properties and the ratio of impinging momentum or velocity to the hydraulic static pressure. The impinging momentum impact is proportional to $\rho U^2 \pi r^2$ and hydraulic "pressure" is proportional to $\rho g h 2 \pi r h$. Where r is the radius of the impinging jet and h is the height of the

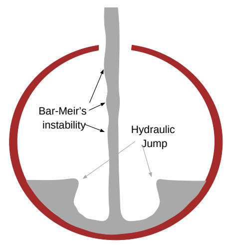

Fig. -3.2. Hydraulic jump in the shot sleeve.

[5]Very few papers (~ 0) can be found dealing with this aspect.

[6]Some elementary estimates of fluid mechanics and heat transfer were made by the author and hopefully will be added to this book.

liquid metal in the sleeve. The above statement leads to

$$U_{critical} \propto \sqrt{\frac{g\,h^2}{r}} \tag{3.1}$$

The critical velocity on the other hand has to be

$$U_critical = g\,h_L \tag{3.2}$$

where h_L is the distance of the ladle to the height of the liquid metal in the sleeve. The height where the hydraulic shock will not exist is

$$h_{critical} \sim \sqrt{r\,h_L} \tag{3.3}$$

This analysis suggests that decreasing the ladle height and/or reducing less mass flow rate (the radius of the jet) result in small critical height. The air entrainment during that time will be discussed in the book "Basic of Fluid Mechanics" in the Multi–Phase flow chapter. At this stage, air bubbles are entrained in the liquid metal which augment the heat transfer. At present, there is an extremely limited knowledge about the heat transfer during this part of the process, and of course less about how to minimize it. However, this analysis suggests that minimizing the ladle height is one of the ways to reduce it.

The heat transfer from liquid metal to the surroundings is affected by the velocity and the flow patterns since the mechanism of heat transfer is changed from a dominated natural convection to a dominated force convection. In addition, the liquid metal jet surface is also affected by heat transfer to some degree by change in the properties.

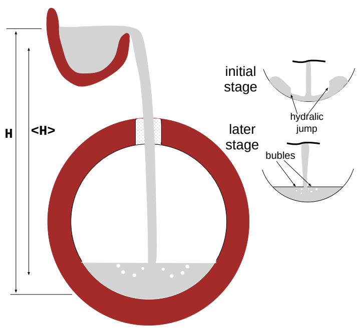

Fig. -3.3. Filling of the shot sleeve.

Heat Transferred to the Jet

The estimate on heat transfer requires some information on jet dynamics. There are two effects that must be addressed; one the average radius and the fluctuation of the radius. As first approximation, the average jet radius changes due to the velocity change. For laminar flow, (for simplicity assume plug flow) the velocity function is $\sim \sqrt{x}$ where x is the distance from the ladle. For constant flow rate, neglecting the change of density, the radius will change as $r \sim 1/\sqrt[4]{x}$. Note that this relationship is not valid when it is very near the ladle proximity ($r/x \sim 0$). The heat transfer increases as a function of x for these two reasons.

The second effect is jet radius fluctuations. Consider this, the jet leaves the ladle in a plug flow. Due to air friction, the shear stress changes the velocity profile to parabolic. For simple assumption of steady state(it is not steady state), the momentum equation which governs the liquid metal is

$$\rho\left(\overbrace{\frac{\partial u_z}{\partial t}}^{\text{assume } 0}+\overbrace{u_r\frac{\partial u_z}{\partial r}}^{\text{constant}}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right] \tag{3.4}$$

Equation (3.4) is in simplified equation form for the gas and liquid phases. Thus, there are two equations that needs to be satisfied simultaneously; one for the gas side and one for the liquid side. Even neglecting several terms for this discussion, it clear that both equations are second order differential equations which have different boundary conditions. Any second order differential equation requires two different boundary conditions. Requirement to satisfy additional boundary condition can be achieved. Thus from physical point of view, second order differential equation which needs to satisfy three boundary conditions is not possible, Thus there must be some wrong either with the governing equation or with the boundary conditions. In this case, the two governing equations must satisfy five (5) different boundary conditions. These boundary conditions are as follows: 1) summitry at $r = 0$, 2) identical liquid metal and air velocities at the interface, 3) identical shear stress at the interface, 4) zero velocity at infinity for the air, and 5) zero shear stress for the air at the infinity. These requirements cannot be satisfied if the interface between the liquid metal and the gas is a straight line.

The heat transfer to the sleeve in the impinging area is significant but at present only very limited knowledge is available due to complexity.

3.2.2 Plunger Slow Moving Part

Fluid Mechanics

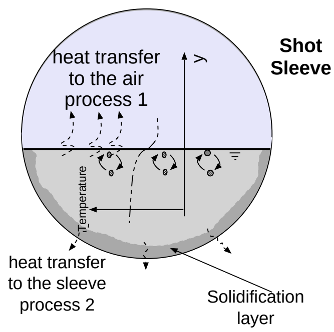

Fig. -3.4. Heat transfer processes in the shot sleeve.

The main point is the estimate for energy dissipation. The dissipation is proportional to $\mu <U>^2 L$. Where the strange velocity, $<U>$ is averaged kinetic velocity provided by jet. This kinetic energy is at most the same as potential energy of liquid metal in the ladle. The potential energy in the ladle is $<H>\ m\,g$ where $<H>$ is averaged height see Figure 3.3. The averaged velocity in the shot sleeve is $\sqrt{2\,g\ <H>}$ The rate energy dissipation can be estimated as $\mu\left(\frac{<U>}{R}\right)^2\ \pi\,R\,L$ as L is the length of shot sleeve. The shear stress is assumed to occur equally in the volume of liquid metal in the sleeve. This assumption of shear stress grossly under estimates the dissipation.

The actual dissipation is larger due to the larger velocity gradients. The estimated time is then

Heat Transfer

In this section, the solidification effects are examined. One of the assumptions in the analysis of the critical slow plunger velocity is that the solidification process does not play an important role (see Figure 3.4). The typical time for heat to penetrate a typical layer in air/gas phase is in the order of minutes. Moreover, the density of the air/gas is 3 order magnitude smaller than liquid metal. Hence, most of the resistance to heat transfer is in the gas phase. Additionally, it has been shown that the liquid metal surface is continuously replaced by slabs of material below the surface which is known in scientific literature as the renewal surface theory. Thus, the main heat transfer mechanism is through the liquid metal to the sleeve. The heat transfer rate for a very thin solidified layer can be approximated as

$$Q \sim k_{lm} \frac{\Delta T}{r} \pi \, r \, L \sim L_s \, \pi \, r \, L \, t \, \rho \tag{3.5}$$

Where L_s is the latent heat, k_{lm} is the thermal conductivity of liquid metal and t is the thickness of the solidification layer. Equation (3.6) results in

$$\frac{t}{r} \propto \frac{k_{lm} \, \Delta T}{L_s \, r^2 \, \rho} \tag{3.6}$$

The value for this die casting process in minutes is in the range of 0.01-0.001 after the thickness reaches to 1-2 [mm]. The relative thickness further decreases as the inverse of the square solidified layer increases. If the solidification is less than one percent of the radius, the speed will be very small compared to the speed of the plunger. If the solidification occur as a mushy zone then the heat transfer is reduced further and it is even lower than this estimate and $\frac{\ell}{R} \ll 1$). Therefore, the heat transfer from the liquid metal surface to the air, as shown in Figure 3.4 (mark as process 1), acts as an insulator to the liquid metal.

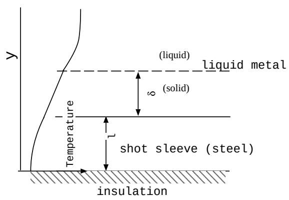

Fig. -3.5. Solidification of the shot sleeve time estimates.

The governing equation in the sleeve is

$$\rho_d {c_p}_d \frac{\partial T}{\partial t} = k_d \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{3.7}$$

where the subscript d denotes the properties of the sleeve material.

Boundary condition between the sleeve and the air/gas is

$$\left. \frac{\partial T}{\partial n} \right|_{y=0} = 0 \tag{3.8}$$

Where n represents the perpendicular direction to the die. Boundary conditions between the liquid metal (solid) and sleeve

$$k_{steel} \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l} = k_{AL} \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l} \tag{3.9}$$

The governing equation for the liquid metal (solid phase)

$$\rho_{lm} c_{p_{lm}} \frac{\partial T}{\partial t} = k_{lm} \left(\frac{\partial^2 T}{\partial y^2} \right) \tag{3.10}$$

where lm denotes the properties of the liquid metal. The dissipation and the velocity are neglected due to the change of density and natural convection.

Boundary condition between the phases of the liquid metal is given by

$$v_s \rho_s h_{sf} = k_l \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l+\delta} - k_s \left(\frac{\partial T}{\partial y} \right) \bigg|_{y=l+\delta} \sim k \frac{\partial (T_l - T_s)}{\partial y} \bigg|_{y=l+\delta} \tag{3.11}$$

h_{sf}	the heat of solidification
ρ_s	liquid metal density at the solid phase
v_n	velocity of the liquid/solid interface
k	conductivity

Neglecting the natural convection and density change, the governing equation in the liquid phase is

$$\rho_l c_{p_l} \frac{\partial T}{\partial t} = k_l \frac{\partial^2 T}{\partial y^2} \tag{3.12}$$

The dissipation function can be assumed to be negligible in this case.

There are three different periods in heat transfer;

1. filling the shot sleeve
2. during the quieting time, and
3. during the plunger movement.

In the first period, heat transfer is relatively very large (major solidification). At present, there is not much known about the fluid mechanics not to say much about the solidification process/heat transfer in fluid mechanics. The second period can be simplified and analyzed as known initial velocity profile. A simplified assumption can be made considering the fact that Pr number is very small (large thermal boundary layer compared to fluid mechanics boundary layer). Additionally, it can be assumed that the natural convection effects are marginal. In the last period, the heat transfer is composed from two zones: 1) behind the jump and 2) ahead of the jump. The heat transfer ahead of the jump is the same as in the second period; while the heat transfer behind the jump is like heat transfer into a plug flow for low Pr number. The heat transfer in such cases have been studied in the past[7].

[7] The reader can refer, for example, to the book "Heat and Mass Transfer" by Eckert and Drake.

3.2.3 Runner system

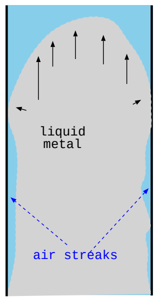

Fig. -3.6. Entrance of liquid metal to the runner.

The flow in the runner system has to be divided into sections; 1) flow with free surface 2) filling the cavity when the flow is pressurized (see Figures 3.6 and 3.7). In the first section the gravity affects the air entrainment. The dominant parameters in this case are Weber number, We and Reynolds number, Re. This phenomenon determines how much metal has to be flushed out. It is well known that the liquid interface cannot be a straight line. Above certain velocity (typical to die casting, high Re number) air leaves streaks of air/gas slabs behind the "front line" as shown in Figure 3.6. These streaks create a low heat transfer zone at the head of the "jet" and "increases" its velocity. The air entrainment created in this case is supposed to be flushed out through the vent system in a proper process design. Unfortunately, at present very little is known about this issue especially the geometry typical to die casting.

Gravity Limited in Runner system

In the second phase, the flow in the runner system is pressurized. The typical velocity is large of the range of 10-15 [m/sec]. The typical runner length is in order of 0.1[m]. The velocity due to gravity is $\approx$ 2.5[m/sec]. The Fr number assumes the value $\sim 10^2$ for which gravity play a limited role.

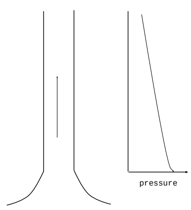

Fig. -3.7. Flow in runner when during pressurizing process.

The converging nozzle such as the transition into runner system (which a good die casting engineer should design) tends to reduce the turbulence, if turbulence exists, and can even eliminate it. In that view, the liquid metal enters the runner system as a laminar flow (actually close to a plug flow). For a duct with a typical dimension of 10 [mm] and a mean velocity, U $= 10$[m/sec], (during the second stage), for aluminum die casting, the Reynolds number is:

$$Re = \frac{Ub}{\nu} \approx 5 \times 10^{-7}$$

which is a supercritical flow. However, the flow is probably laminar flow due to the short time.

Another look at turbulence issue: The boundary layer is a function of the time (during the filling period) is of order

$$\delta = 12\nu t$$

The boundary layer in this case can be estimated as[8] the time of the first phase. Anyhow, utilizing the time of 0.01[sec] the viscosity of aluminum in the boundary layer is of the thickness of 0.25[mm] which indicates that flow is laminar.

3.2.4 Die Cavity

All the numerical simulations of die filling are done almost exclusively by assuming that the flow is turbulent and continuous (no two phase flow). In the section 3.3.1 a question about the question whether existence of turbulence is discussed and if so what kind of model is appropriate. Thus, the validity of these numerical models is examined. The liquid metal enters the cavity as a non–continuous flow. According to some researchers, it is preferred that the flow will be atomized (spray). While there is a considerable literature about many geometries none available to typical die casting configurations[9]. The flow can be atomized as either in laminar or turbulent region. The experiments by the author and by others, showed that the flow turns into spray in many cases (See Figures 3.8).

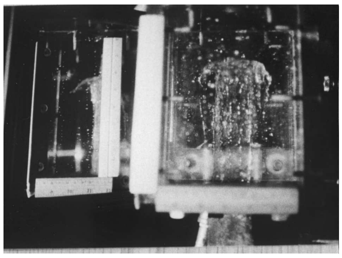

Fig a. Flow as a jet.

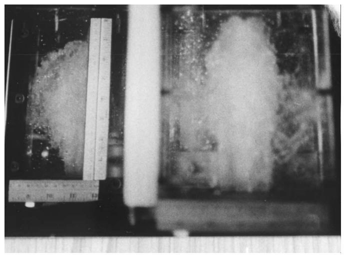

Fig b. Flow as a spray.

Fig. -3.8. Typical flow pattern in die casting, jet entering into empty cavity.

In the section 3.4.1 it was shown that the time for atomization is very fast compared with any other process (filling time scale and, of course, the conduction heat transfer or solidification time scales). Atomization requires two streams with a significant velocity difference; stronger surface tension forces against the maintaining stability forces. Numerous experimental studies have shown that better castings are obtained when the injected velocity is above a certain value. This fact alone is enough to convince researchers that the preferred flow pattern is a spray flow. Yet, only a very small number of numerical models exist assuming spray flow and are used for die casting (for example, the paper by Hu at el [22].). Experimental work commonly cited as a "proof"

[8] only during the flow in the runner system, no filling of the cavity

[9] One can just wonder who were the opposition to this research? Perhaps one of the referees as in the Appendix B for the all clues that have been received.

of turbulence was conducted in the mid 60s [30] utilizing water analogy[10]. The "white" spats they observed in their experiments are atomization of the water. Because these experiments were poorly conducted (no similarity to die casting process) the observation/information from these studies is very limited. Yet with this limitation in mind, one can conclude that the spray flow does exist.

Experiments by Fondse et al [16] show that atomization is larger in laminar flow compared to a turbulent flow in a certain range. This fact further creates confusion of what is the critical velocity needed in die casting. Since the experiments which measure the critical velocity were poorly conducted, no reliable information is available on what is the flow pattern and what is the critical velocity[11].

3.2.5 Intensification Period

The two main concerns in this phase is to extract heat from the die and to solidify the liquid metal as aptly as possible to obtain the final shape. Thus, two operational parameters are important; one the (minimum) time for the intensification and two the pressure of the intensification (the clamping force). These two operational parameters can improve casting design to obtain good product.

The main resistance to the heat flow is in the die and the cooling liquid (oil or water based solution). In some parts of the process, the heat is transformed to the cooling liquid via the boiling mechanism. However, the characteristic of boiling heat transfer time to achieve a steady state is larger than the whole process and the typical equations (steady state) for the preferred situation (heat transfer only in the first mode) are not accurate. When there is very limited understanding of so many aspects of the process, the effects of each process on other processes are also cluttered.

3.3 Special Topics

3.3.1 Is the Flow in Die Casting Turbulent?

Transition from laminar to turbulent

It is commonly assumed that the flow in die casting processes is turbulent in the shot sleeve, runner system, and during the cavity filling. Further, it also assumed that the $k-\epsilon$ model can reasonably represent

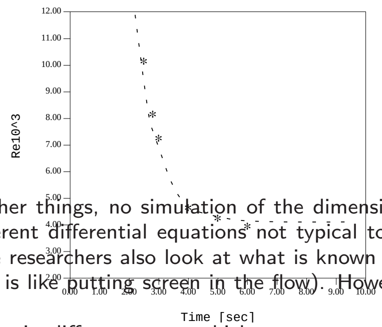

Fig. -3.9. Transition to turbulent flow in circular pipe for instantaneous flow after Wygnanski and others by interpolation.

[10] The problems in these experiments were, among other things, no simulation of the dimensional numbers such as Re, Geometry etc. and therefore different differential equations not typical to die casting were "solved." ??punctuation inside quotes] The researchers also look at what is known as a "poor design" for disturbances to flow downstream (this is like putting screen in the flow). However, a good design requires smooth contours.

[11] Beside other problems such as different flow velocity in different gates which were never really measured, the pressure in the cavity and quality of the liquid metal entering the cavity (is it in two phase?) were never recorded.

the turbulence structure. These assumptions are examined herein. The flow can be examined in three zones: 1) the shot sleeve, 2) the runner system, and 3) the mold cavity. Note, even if the turbulence exists in some regions, it doesn't necessarily mean that all the flow field is turbulent.

Is the flow in the shot sleeve turbulent as the EKK sale engineers claim? These sale engineers did not present any evidence or analysis for such claims. For a simple analysis, the initial part of the shot sleeve filling, the liquid metal goes through a hydraulic jump. The flow after the hydraulic jump is very slow because the increase of the ratio of cross section areas. For example, casting of the 1[kg] from height of 0.2[m] to a shot sleeve of 0.1[m] creates a velocity in shot sleeve of $\sqrt{2}[m/sec]$ which results after the hydraulic jump to be with velocity about 0.01[m/sec]. The Reynolds number for this velocity is $\sim 10^4$ and Froude number of about 10. After the jump the Froude number is reduced and the flow is turbulent. However, by the time the hydraulic jump vanishes, the flow turns into laminar flow and no change (waviness) in the surface can be observed. It can be noticed that the time scale for the dissipation is about the same scale as the time for the operation of the next stage.

Figure 3.9 exhibits the transition to a turbulent flow for instantaneous starting flow in a circular pipe. The abscissa represents time and the y–axis represents the Re number at which transition to turbulence occurs. The points on the graphs show the transition to a turbulence. This figure demonstrates that a large time is required to turn the flow pattern to turbulent which is measured in several seconds. The figure demonstrates that the transition does not occur below a certain critical Re number (known as the critical Re number for steady state). It also shows that a considerable time has elapsed before transition to turbulence occurs even for a relatively large Reynolds number. The geometry in die casting however is different and therefore it is expected that the transition occurs at different times. Our present knowledge of this area is very limited. Yet, a similar transition delay is expected to occur after the "instantaneous" start–up which probably will be measured in seconds. The flow in die casting in many situations is very short (in order of milliseconds) and therefore it is expected that the transition to a turbulent flow does not occur.

After the liquid metal is poured, it is normally repose for sometime in a range of 10 seconds. This fact is known in the scientific literature as the quieting time for which the existed turbulence (if exist) is reduced and after enough time (measured in seconds) is illuminated. Hence, the turbulence, which was created during the filling process of the shot sleeve, 'disappear'

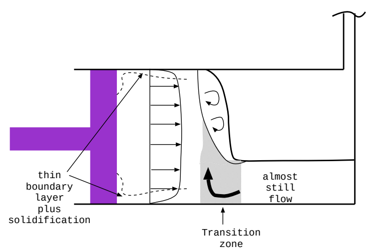

Fig. -3.10. Flow pattern in the shot sleeve.

due to viscous dissipation. The question is, whether the flow in the duration of the slow plunger velocity turbulent (see Figure 3.10) can be examined.

Clearly, the flow in the substrate (a head of the wave) is still (almost zero velocity) and therefore the turbulence does not exist. The Re number behind the wave is above the critical Re number (which is in the range of 2000–3000). The typical time for the wave to travel to the end of the shot sleeve is in the range of a $\sim 10^0$ second. At present there are no experiments on the flow behind the wave[12]. The estimation can be done by looking at what is known in the literature about the transition to turbulence in instantaneous starting pipe flow. It has been shown [32] that the flow changes from laminar flow to turbulent flow in an abrupt manner for a flow with supercritical Re number.

A typical velocity of the propagating front (transition between laminar to turbulent) is about the same velocity as the mean velocity of the flow. Hence, it is reasonable to assume that the turbulence is confined to a small zone in the wave front since the wave is traveling in a faster velocity than the mean velocity. Note that the thickness of the transition layer is a monotone increase function of time (traveling distance). The Re number in the shot sleeve based on the diameter is in a range of $\sim 10^4$ which means that the boundary layer has not developed much. Therefore, the flow can be assumed as almost a plug flow with the exception of the front region.

A Note on Numerical Simulations

The most common model for turbulence that is used in the die casting industry for simulating the flow in cavity is $k - -\epsilon$. This model is based on several assumptions

1. isentropic homogeneous turbulence,
2. constant material properties (or a mild change of the properties),
3. continuous medium (only liquid (or gas), no mixing of the gas, liquid and solid whatsoever), and
4. the dissipation does not play a significant role (transition to laminar flow).

The $k - -\epsilon$ model is considered reasonable for the cases where these assumptions are not far from reality. It has been shown, and should be expected, that in cases where assumptions are far from reality, the $k - -\epsilon$ model produces erroneous results. Clearly, if we cannot determine whether the flow is turbulent and in what zone, the assumption of isentropic homogeneous turbulence is very questionable. Furthermore, if the change to turbulence just occurred, one cannot expect the turbulence to have sufficient time to become isentropic homogeneous. As if this is not enough complication, consider the effects of properties variations as a result of temperature change. Large variations of the properties such as the viscosity have been observed in many alloys especially in the mushy zone.

[12]It has to be said that similar situations are found in two phase flow but they are different by the fact the flow in two phase flow is a sinusoidal in some respects.

While the assumption of the continuous medium is semi reasonable in the shot sleeve and runner, it is far from reality in the die cavity. As discussed previously, the flow is atomized and it is expected to have a large fraction of the air in the liquid metal and conversely some liquid metal drops in the air/gas phase. In such cases, the isentropic homogeneous assumption is very dubious.

For these reasons the assumption of $k - -\epsilon$ model seems unreasonable unless good experiments can show that the choice of the turbulence model does not matter in the calculation.

The question whether the flow in die cavity is turbulent or laminar is secondary. Since the two phase flow effects have to be considered such as atomization, air/gas entrainment etc. to describe the real flow in the cavity.

Additional note on numerical simulation

The solution of momentum equation for certain situations may lead to unstable solution. Such case is the case of two jets with different velocity flow into a medium and they are adjoined (see Figure 3.11). The solution of such flow can show that the velocity field can be an unstable solution for which the flow moderately changes to become like wave flow. However, in many cases this flow can turn out to be full with vortexes and such. The reason that this happened is the introduction of instabilities. Numerical calculations intrinsically are introducing instabilities because of truncation of the calculations. In many cases, these truncations results in over–shooting or under–shooting of the nature instability. In cases where the flow is unstable, a careful study is required to make sure that the solution did not produce an unrealistic solution for larger or smaller than reality introduced instabilities. An excellent example of such poor understating is a work made in EKK company [2]. In that work, the flow in the shot sleeve was analyzed. The nature of the flow is two dimensional which can be seen by all the photos taken by numerous people (staring from the 50s). The presenter of that work explained that they have used 3D calculations because they want to study the instabilities perpendicular to the flow direction. The numerical "instability" in this case is larger than real instabilities and therefore, the numerical results show phenomena does not exist in reality.

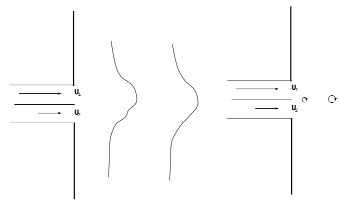

Fig. -3.11. Two streams of fluids into a medium.

Reverse transition from turbulent flow to laminar flow

After filling the die cavity, during the solidification process and intensification, the attained turbulence (if exist) is reduced and probably eliminated, i.e. the flow is laminar in a large portion of the solidification process. At present we don't comprehend when the transition point/criteria occurs and we must resort to experiments. It is a hope that some real good experiments using the similarity technique, outlined in this book, will be performed. So more knowledge can be gained and hopefully will appear in this book.

3.3.2 Dissipation effect on the temperature rise

The large velocities of the liquid metal (particularly at the runner) theoretically can increase the liquid metal temperature. To study this phenomenon, compare the of maximum effect of all the kinetic energy that is transformed into thermal energy.

$$\frac{U^2}{2} = c_p \Delta T \tag{3.13}$$

This equation leads to the definition of Eckert number

$$Ec = \frac{U^2}{c_p \Delta T} \tag{3.14}$$

When Ec number is very large it means that the dissipation plays a significant role and conversely when Ec number is small the dissipation effects are minimal. In die casting, Eckert number, Ec, is very small therefore the thermal dissipation is very small and can be ignored.

3.3.3 Gravity effects

The gravity has a large effect only when the gravity force is large relatively to other forces. A typical velocity range generated by gravity is the same as for an object falling through the air. The air effects can be neglected since the air density is very small compared with liquid metal density. The momentum is the other dominate force in the filling of the cavity. Thus, the ratio of the momentum force to the gravity force, also known as Froude number, determines if the gravity effects are important. The Froude number is defined here as

$$Fr = \frac{U^2}{\ell g} \tag{3.15}$$

Where U is the velocity, ℓ is the characteristic length g is the gravity force. For example, the characteristic pouring length is in order of 0.1[m], in extreme cases the velocity can reach 1.6[m] with characteristic time of 0.1[sec]. The author is not aware of experiments to verify the flow pattern in such cases (low Pr number due to solidification effect)[13] Yet, it is reasonable to assume that the liquid metal in such a case, flow in laminar regimes even though the Re number is relatively large ($\sim 10^4$) because of the short time and the short distance. The Re number is defined by the flow rate and the thickness of the exiting typical dimension. Note, the velocity reached its maximum value just before impinging on the sleeve surface.

[13] It be interesting to find such experiments.

The gravity has dominate effects on the flow in the shot sleeve since the typical value of the Froude number in that case (especially during the slow plunger velocity period) is in the range of one(1). Clearly, any analysis of the flow has to take into consideration the gravity (see Chapter 8).

3.4 Estimates of the time scales in die casting

3.4.1 Utilizing semi dimensional analysis for characteristic time

The characteristic time scales determine the complexity of the problem. For example, if the time for heat transfer/solidification process in the die cavity is much larger than the filling time, then the problem can be broken into three separate cases 1) the fluid mechanics, the filling process, 2) the heat transfer and solidification, and 3) dissipation (maybe considered with solidification). Conversely, the real problem in die filling is that we would like for the heat transfer process to be slower than the filling process, to ensure a proper filling. The same can be said about the other processes.

filling time

The characteristic time for filling a die cavity is determined by

$$t_f \sim \frac{L}{U} \tag{3.16}$$

Where L denotes the characteristic length of the die and U denotes the average filling velocity, determined by the pQ^2 diagram, in most practical cases this time typically is in order of 5–100 [millisecond]. Note, this time is not the actual filling time but related to it.

Atomization time

The characteristic time for atomization for a low Re number (large viscosity) is given by

$$t_{a_{viscosity}} = \frac{\nu \ell}{\sigma} \tag{3.17}$$

where ν is the kinematic viscosity, σ is the surface tension, and ℓ is the thickness of the gate. The characteristic time for atomization for large Re number is given by

$$t_{a_{momentum}} = \frac{\rho \ell^2 U}{\sigma} \tag{3.18}$$

The results obtained from these equations are different and the actual atomization time in die casting has to be between these two values.

Conduction time (die mold)

The governing equation for the heat transfer for the die reads

$$\rho_d {c_p}_d \frac{\partial T_d}{\partial t} = k_d \left(\frac{\partial^2 T_d}{\partial x^2} + \frac{\partial^2 T_d}{\partial y^2} + \frac{\partial^2 T_d}{\partial z^2} \right) \tag{3.19}$$

To obtain the characteristic time we dimensionless–ed the governing equation and present it with a group of constants that determine value of the characteristic time by setting it to unity. Denoting the following variables as

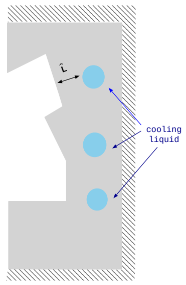

Fig. -3.12. Schematic of heat transfer processes in the die.

$$t_d^{'} = \frac{t}{t_{c_d}} \qquad x_d^{'} = \frac{x}{\tilde{L}}$$

$$y_d^{'} = \frac{y}{\tilde{L}} \qquad z_d^{'} = \frac{z}{\tilde{L}} \qquad \theta_d = \frac{T - T_B}{T_M - T_B} \tag{3.20}$$

$\tilde{L}$ the characteristic path of the heat transfer from the die inner surface to the cooling channels

subscript

B boiling temperature of cooling liquid

M liquid metal melting temperature

With these definitions, equation (3.19) is transformed to

$$\frac{\partial \theta_d}{\partial t} = \frac{t_{c_d} \alpha_d}{\tilde{L}^2} \left(\frac{\partial^2 \theta_d}{\partial x'^2} + \frac{\partial^2 \theta_d}{\partial y'^2} + \frac{\partial^2 \theta_d}{\partial z'^2} \right) \tag{3.21}$$

which leads into estimate of the characteristic time as

$$t_{c_d} \sim \frac{\tilde{L}^2}{\alpha_d} \tag{3.22}$$

Note the characteristic time is not effected by the definition of the θ_d.

Conduction time in the liquid metal (solid)

The governing heat equation in the solid phase of the liquid metal is the same as equation (3.19) with changing properties to liquid metal solid phase. The characteristic time for conduction is derived similarly as done previously by introducing the dimensional parameters

$$t' = \frac{t}{t_{c_s}}; \; x' = \frac{x}{\ell}; \; y' = \frac{y}{\ell}; \; z' = \frac{z}{\ell}; \;\; \theta_s = \frac{T - T_B}{T_M - T_B} \tag{3.23}$$

where t_{c_s} is the characteristic time for conduction process and, ℓ, denotes the main path of the heat conduction process die cavity. With these definitions, similarly as was done before the characteristic time is given by

$$t_{c_s} \sim \frac{\ell^2}{\alpha_s} \tag{3.24}$$

Note again that α_s has to be taken for properties of the liquid metal in the solid phase. Also note that the solidified length, ℓ, changes during the process and discussing the case where the whole die is solidified is not of interest. Initially the thickness, $\ell = 0$ (or very small). The characteristic time for very thin layers is very small, $t_{c_s} \sim 0$. As the solidified layer increases the characteristic time also increases. However, the temperature profile is almost established (if other processes were to remain in the same conditions). Similar situations can be found when a semi infinite slab undergoes solidification with ΔT changes as well as results of increase in the resistance. For the foregoing reasons the characteristic time is very small.

Solidification time

Miller's approach

Following Eckert's work, Miller and his student [20] altered the calculations[14] and based the assumption that the conduction heat transfer characteristic time in die (liquid metal in solid phase) is the same order magnitude as the solidification time. This assumption leads them to conclude that the main resistance to the solidification is in the interface between the die and mold [15]. Hence they conclude that the solidified front moves according to the following

$$\rho h_{sl} v_n = h \Delta T \tag{3.25}$$

Where here h is the innovative heat transfer coefficient between solid and solid[16] and v_n is front velocity. Then the filling time is given by the equation

$$t_s = \frac{\rho h_{sl}}{h \Delta T} \ell \tag{3.26}$$

[14] Miller and his student calculate the typical forces required for clamping. The calculations of Miller has shown an interesting phenomenon in which small casting (2[kg]) requires a larger force than heavier casting (20[kg])?! Check it out in their paper, page 43 in NADCA Transaction 1997! If the results extrapolated (not to much) to about 50[kg] casting, no force will be required for clamping. Furthermore, the force for 20 [kg] casting was calculated to be in the range of $4000[N]$. In reality, this kind of casting will be made on 1000 [ton] machine or more (3 order of magnitude larger than Miller calculation suggested). The typical required force should be determined by the plunger force and the machine parts transient characteristics etc. Guess, who sponsored this research and how much it cost!

[15] An example how to do poor research. These kind of research works are found abundantly in Dr. Miller and Dr. J. Brevick from Ohio State Univerity. These works when examined show contractions with the logic and the rest of the world of established science.

[16] This coefficient is commonly used either between solid and liquid, or to represent the resistance between two solids. It is hoped that Miller and coworkers refer that this coefficient to represent the resistance between the two solids since it is a minor factor and does not determine the characteristic time.

where ℓ designates the half die thickness. As a corollary conclusion one can arrive from this construction is that the filling time is linearly proportional to the die thickness since $\rho h_{sl}/h\Delta T$ is essentially constant (according to Miller). This interesting conclusion contradicts all the previous research about solidification problem (also known as the Stefan problem). That is if h is zero the time is zero also. The author is not aware of any solidification problem to show similar results. Of course, Miller has all the experimental evidence to back it up!

Present approach

Heat balance at the liquid-solid interface yields

$$\rho_s h_{sf} v_n = k\frac{\partial (T_l - T_s)}{\partial n} \tag{3.27}$$

where n is the direction perpendicular to the surface and ρ has to be taken at the solid phase see Appendix 10. Additionally note that in many alloys, the density changes during the solidification and is substantial which has a significant effect on the moving of the liquid/solid front. It can be noticed that at the die interface $k_s\partial T/\partial n \cong k_d \partial T/\partial n$ (opposite to Miller) and further it can be assumed that temperature gradient in the liquid side, $\partial T/\partial n \sim 0$, is negligible compared to other fluxes. Hence, the speed of the solid/liquid front moves

$$v_n = \frac{k}{\rho_s h_{sl}}\frac{\partial T_s - T_l}{\partial n} \sim \frac{k\Delta T_{MB}}{\rho_s h_{sl}\tilde{L}} \tag{3.28}$$

Notice the difference to equation (3.26) The main resistance to the heat transfer from the die to the mold (cooling liquid) is in the die mold. Hence, the characteristic heat transfer from the mold is proportional to $\Delta T_{MB}/\tilde{L}$[17]. The characteristic temperature difference is between the melting temperature and the boiling temperature. The time scale for the front can be estimated by

$$t_s = \frac{\ell}{v_s} = \frac{\rho_s h_{sl}\ell^2\left(\frac{\tilde{L}}{\ell}\right)}{k_d\Delta T_{MB}} \tag{3.29}$$

Note that the solidification time isn't a linear function of the die thickness, ℓ, but a function of $\sim \left(\ell^2\right)$[18].

Dissipation Time

Examples of how dissipation is governing the flow can be found abundantly in nature.

$$\left(\frac{\partial\theta_l}{\partial t} + u\frac{\partial\theta_l}{\partial x} + v\frac{\partial\theta_l}{\partial y} + w\frac{\partial\theta_l}{\partial z}\right) = \alpha_l\left(\frac{\partial^2\theta_l}{\partial x^2} + \frac{\partial^2\theta_l}{\partial y^2} + \frac{\partial^2\theta_l}{\partial z^2}\right) + \mu\Phi \tag{3.30}$$

[17] The estimate can be improved by converting the resistances of the die to be represented by die length and the same for the other resistance into the cooling liquid i.e. $\Sigma 1/h_o + \tilde{L}/k + C_D ots + 1/h_i$.

[18] $\tilde{L}$ can be represented by ℓ for example, see more simplified assumption leads to pure $= \ell^2$.

Where Φ the dissipation function is defined as

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left[\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right]^2 +$$
$$\left[\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right]^2 + \left[\frac{\partial w}{\partial y}\right) + \left(\frac{\partial v}{\partial z}\right]^2 - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 \tag{3.31}$$

Since the dissipation characteristic time isn't commonly studied in "regular" fluid mechanics, we first introduce two classical examples of dissipation problems. First problem deals with the oscillating manometer and second problem focuses on the "rigid body" brought to a rest in a thin cylinder.

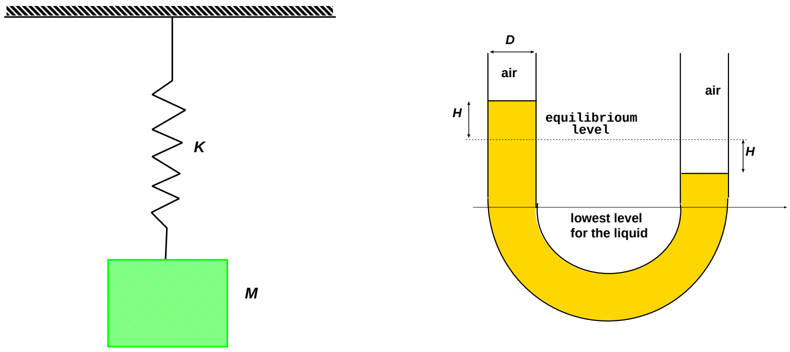

Fig a. Mass, spring

Fig b. Oscillating manometer

Fig. -3.13. The oscillating manometer for the example 3.1.

Example 3.1:
A liquid in manometer is disturbed from a rest by a distance of H_0. Assume that the flow is laminar and neglected secondary flows. Describe $H(t)$ as a function of time. Defined 3 cases: 1)under damping, 2) critical damping, and 3) over damping. Discuss the physical significance of the critical damping. Compute the critical radius to create the critical damping. For simplicity assume that liquid is incompressible and the velocity profile is parabolic.

<u>SOLUTION</u>

The conservation of the mechanical energy can be written as

$$\frac{d}{dt}\begin{pmatrix}\text{rate of increase}\\\text{of kinetic and}\\\text{potential energy}\\\text{in system}\end{pmatrix} = \Delta\begin{pmatrix}\text{total of inflow of}\\\text{kinetic energy}\end{pmatrix} + \Delta\begin{pmatrix}\text{total of inflow of}\\\text{potential energy}\end{pmatrix} +$$
$$\Delta\begin{pmatrix}\text{total of inflow of}\\\text{potential energy}\end{pmatrix} + \Delta\begin{pmatrix}\text{total net rate of}\\\text{surroundings}\\\text{work on the}\\\text{system}\end{pmatrix} + \Delta\begin{pmatrix}\text{total work due}\\\text{to expansion or}\\\text{compression of}\\\text{fluid}\end{pmatrix} + \Delta\begin{pmatrix}\text{total rate}\\\text{mechanical}\\\text{energy}\\\text{dissipated}\\\text{because}\\\text{viscosity}\end{pmatrix} \tag{3.32}$$

The chosen system is the liquid in the manometer. There is no flow in or out of the liquid of the manometer, and thus, terms that deal with flow in or out are canceled. It is assumed that the surface at the interface is straight without end effects like surface tension. This system is unsteady and therefore the velocity profile is function of the time and space. In order to demonstrate the way the energy dissipation is calculated it is assumed the velocity is function of the radius and time but separated. This assumption is wrong and cannot be used for real calculations because the real velocity profile is not separated and can have positive and negative velocities. It is common to assume that velocity profile is parabolic which is for the case where steady state is obtained.

This assumption can used as a limiting case and the velocity profile is

$$U(r,t) = U(r) = U_0(t)\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{3.33}$$

where R the radius of the manometer. The velocity at the center is a function of time but independent of the Length. It can be noticed that this equation dim:eq:velocityH is problematic because it breaks the assumption of the straight line of the interface.

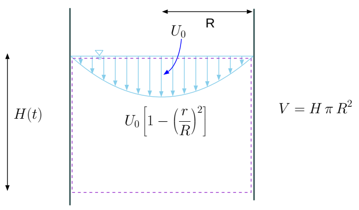

Fig. -3.14. Mass Balance to determine the relationship between the U_0 and the Height, H.

The relationship between the velocity at the center, U_0 to the height, $H(t)$ can be obtained from mass conservation on left side of the manometer (see Figure 3.14) is

$$\frac{d\left(\not\rho\, H\,\pi\,R^2\right)}{dt} = \int_0^R \not\rho\, U_0 \left[1 - \left(\frac{r}{R}\right)^2\right] \overbrace{2\,\pi\,r\,dr}^{dA} \tag{3.34}$$

Equation (3.34) relates $H(t)$ to the center velocity, U_0, and the integration results in

$$\frac{d\,H}{dt} = \frac{U_0}{2} \tag{3.35}$$

Note that $H(t)$ isn't a function of the radius, R. This relationship (3.35) is based on the definition that U_0 is positive for the liquid flowing to right and therefore the height decreases. The total kinetic energy in the tube is then

$$K_k = \int_0^L \int_0^R \frac{\rho\,{U_0}^2}{2}\left[1 - \left(\frac{r}{R}\right)^2\right]^2 \overbrace{2\,\pi\,r\,dr}^{dA}\,d\ell = \frac{L\,{U_0}^2\,\pi\,R^2}{6} \tag{3.36}$$

where L is the total length (from one interface to another) and $d\ell$ is a coordinate running along the axis of the manometer neglecting the curvature of the "U" shape. It can be noticed that L is constant for incompressible flow. It can be observed that the disturbance of the manometer creates a potential energy which can be measured from a datum at the maximum lower point. The maximum potential energy is obtained when H is either maximum or minimum. The maximum kinetic energy is obtained when H is zero. Thus, at maximum height, H_0 the velocity is zero. The total potential of the system is then

$$K_p = \overbrace{\int_0^{H_0-H} (\rho\, g\, \ell)\, \overbrace{\pi\, R^2 d\ell}^{dV}}^{\text{left side}} + \overbrace{\int_0^{H_0+H} (\rho\, g\, \ell)\, \overbrace{\pi\, R^2 d\ell}^{dV}}^{\text{right side}} = \left({H_0}^2 + H^2\right) \rho\, g\, \pi\, R^2 \tag{3.37}$$

The last term to be evaluated is the viscosity dissipation. Based on the assumptions in the example, the velocity profile is function only of the radius thus the only gradient of the velocity is in the r direction. Hence

$$E_d = \mu\, \Phi = L\, \mu \int_0^R \left(\frac{dU}{dr}\right)^2 \overbrace{2\, \pi\, r\, dr}^{dA} \tag{3.38}$$

The velocity derivative can be obtained by using equation (3.33) as

$$\frac{dU}{dr} = U_0 \left(\frac{-2\, r}{R^2}\right) \Longrightarrow \left(\frac{dU}{dr}\right)^2 = \left(\frac{4\, r^2\, {U_0}^2}{R^4}\right) \tag{3.39}$$

Substituting equation (3.39) into equation (3.37) reads

$$E_d = \mu\, 2\, \pi\, L \int_0^R \overbrace{\frac{4\, r^2\, {U_0}^2}{R^2}}^{\left(\frac{dU}{dr}\right)^2} \frac{r}{R} \frac{dr}{R} = 2\, \pi\, L\, \mu\, R^2\, {U_0}^2 \tag{3.40}$$

The work done on system is neglected by surroundings via the pressure at the two interfaces because the pressure is assumed to be identical. Equation (3.32) is transformed, in this case, into

$$\frac{d}{dt} (K_k + K_p) = -E_d \tag{3.41}$$

The kinetic energy derivative with respect to time (using equation (3.35)) is

$$\frac{d\, K_k}{dt} = \frac{d}{dt} \left(\frac{L\, {U_0}^2\, \pi\, R^2}{6}\right) = \frac{L\, \pi\, R^2}{6} 2\, U_0\, \frac{U_0}{dt} = \frac{4\, L\, \pi\, R^2}{3} \frac{d\, H}{dt} \frac{d^2\, H}{dt^2} \tag{3.42}$$

The potential energy derivative with respect to time is

$$\frac{d\, K_p}{dt} = \frac{d}{dt} \left[\left({H_0}^2 + H^2\right) \rho\, g\, \pi\, R^2\right] = 2\, H\, \frac{dH}{dt}\, \rho\, g\, \pi\, R^2 \tag{3.43}$$

Substituting equations (3.43), (3.42) and (3.40) into equation (3.41) results in

$$\frac{4\,L\,\pi\,R^2}{3}\frac{d\,H}{dt}\,\frac{d^2\,H}{dt^2} + 2\,H\,\frac{dH}{dt}\,\rho\,g\,\pi\,R^2 + 2\,\pi\,L\,\mu\,{U_0}^2 = 0 \tag{3.44}$$

Equation (3.45) can be simplified using the identity of (3.35) to be

$$\frac{d^2\,H}{dt^2} + \frac{6\,\mu}{\rho\,R^2}\frac{dH}{dt} + \frac{3\,g}{2\,L}H = 0 \tag{3.45}$$

This equation is similar to the case mass tied to a spring with damping. This equation is similar to RLC circuit[19]. The common method is to assume that the solution of the form of $A\,e^{\xi\,t}$ where the value of A and ξ will be such determined from the equation. When substituting the "guessed" function into result that ξ having two possible solution which are

$$\xi = \frac{-\frac{6\,\mu}{\rho\,R^2} \pm \sqrt{\left(\frac{6\,\mu}{\rho\,R^2}\right)^2 - \frac{6\,g}{L}}}{2} \tag{3.46}$$

Thus, the solution is

$$\begin{array}{ccc} H = A\,e^{\xi_1\,t} + A\,e^{\xi_2\,t} & \Longrightarrow & \xi_1 \neq \xi_2 \\ H = A\,e^{\xi\,t} + A\,e^{\xi\,t} & \Longrightarrow & \xi_1 = \xi_2 = \xi \end{array} \tag{3.47}$$

The constant A_1 and A_2 are to be determined from the initial conditions. The value under the square root determine the kind of motion. If the value is positive then the system is over–damped and the liquid height will slowly move the equilibrium point. If the value in square is zero then the system is referred to as critically damped and height will move rapidly to the equilibrium point. If the value is the square root is negative then the solution becomes a combination of sinuous and cosines. In the last case the height will oscillate with decreasing size of the oscillation. The critical radius is then

$$R_c = \sqrt[4]{\frac{6\,\mu^2\,L}{g\,\rho^2}} \tag{3.48}$$

It can be observed that this analysis is only the lower limit since the velocity profile is much more complex. Thus, the dissipation is much more significant.

End solution

Example 3.2:

A thin ($t/D \ll 1$) cylinder full with liquid is rotating in a velocity, ω. The rigid body

[19] An electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel.

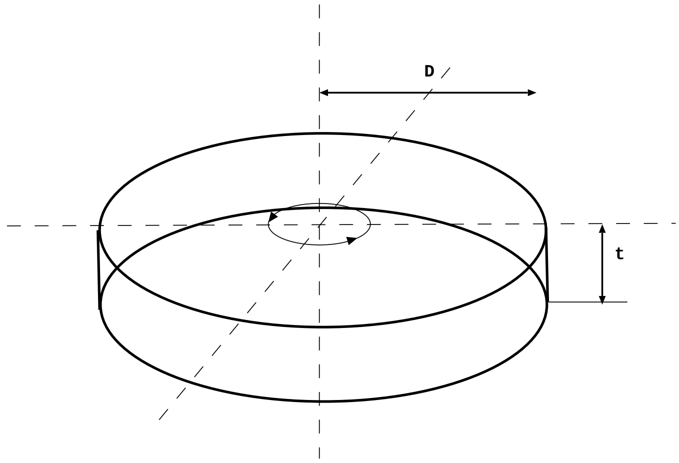

Fig. -3.15. Rigid body brought into rest.

is brought to a stop. Assuming no secondary flows (Bernard's cell, etc.), describe the flow as a function of time. Utilize the ratio $1 \gg t/D$.

$$\frac{d^2X}{dt^2} + \left(\frac{\mu}{\ell^2}\right)\frac{dX}{dt} + X = 0 \tag{3.49}$$

Discuss the case of rapid damping, and the case of the characteristic damping

<u>SOLUTION</u>

End solution

These examples illustrate that the characteristic time of dissipation can be assessed by $\sim \mu(du/d``y'')^2$ thus given by ℓ^2/ν. Note the analogy between t_s and t_{diss}, for which ℓ^2 appears in both of them, the characteristic length, ℓ, appears as the typical die thickness.

3.4.2 The ratios of various time scales

The ratio of several time ratios can be examined for typical die casting operations. The ratio of solidification time to the filling time

$$\frac{t_f}{t_s} \sim \frac{Lk_d\Delta T_{MB}}{U\rho_s h_{sl}\ell\tilde{L}} = \frac{Ste}{Pr\ Re}\left(\frac{\rho_{lm}}{\rho_s}\right)\left(\frac{k_d}{k_{lm}}\right)\left(\frac{L}{\tilde{\tilde{L}}}\right) \tag{3.50}$$

where

Re Reynolds number $\frac{U\ell}{\nu_{lm}}$

Ste Stefan number $\frac{c_{p_{lm}} \Delta T_{MB}}{h_{sl}}$

the discussion is augmented on the importance of equation (3.50). The ratio is extremely important since it actually defines the required filling time.

$$t_f = C \left(\frac{\rho_{lm}}{\rho_s}\right) \left(\frac{k_d}{k_{lm}}\right) \left(\frac{L}{\tilde{L}}\right) \frac{Ste}{Pr\ Re} \tag{3.51}$$

At the moment, the "constant", C, is unknown and its value has to come out from experiments. Furthermore, the "constant" is not really a constant and is a very mild function of the geometry. Note that this equation is also different from all the previously proposed filling time equations, since it takes into account solidification and filling process[20].

The ratio of liquid metal conduction characteristic time to characteristic filling time is given by

$$\frac{t_{c_d}}{t_f} \sim \frac{U\tilde{L}^2}{L\alpha} = \frac{U\ell}{\nu} \frac{\nu}{\alpha} \frac{\tilde{L}^2}{L\ell} = Re\ Pr \frac{\tilde{L}^2}{L\ell} \tag{3.52}$$

The solidification characteristic time to conduction characteristic time is given by

$$\frac{t_s}{t_c} \sim \frac{\rho_s h_{sl} \ell \tilde{L} \alpha_d}{k_d \Delta T_{MB} \tilde{L}^2} = \frac{1}{Ste} \left(\frac{\rho_s}{\rho_d}\right) \left(\frac{c_{p_{lm}}}{c_{p_d}}\right) \left(\frac{\ell}{\tilde{L}}\right) \tag{3.53}$$

The ratio of the filling time and atomization is

$$\frac{t_{a_{viscosity}}}{t_f} \approx \frac{\nu\ell U}{\sigma L} = Ca \left(\frac{\ell}{L}\right) \sim 6 \times 10^{-8} \tag{3.54}$$

Note that ℓ, in this case, is the thickness of the gate and not of the die cavity.

$$\frac{t_{a_{momentum}}}{t_f} \approx \frac{\rho\ell^2 U^2}{\sigma L} = We \left(\frac{\ell}{L}\right) \sim 0.184 \tag{3.55}$$

which means that if atomization occurs, it will be very fast compared to the filling process.

The ratio of the dissipation time to solidification time is given by

$$\frac{t_{diss}}{t_s} \sim \frac{\ell^2}{\nu_{lm}} \frac{k_d \Delta T_{MB}}{\rho_s h_{sl} \ell \tilde{L}} = \left(\frac{Ste}{Pr}\right) \left(\frac{k_d}{k_{lm}}\right) \left(\frac{\rho_{lm}}{\rho_s}\right) \left(\frac{\ell}{\tilde{L}}\right) \sim 10^0 \tag{3.56}$$

this equation yields typical values for many situations in the range of 10^0 indicating that the solidification process is as fast as the dissipation. It has to be noted that when the solidification progress, the die thickness decreases. The ratio, $\ell/\tilde{L}$, reduced as well. As a result, the last stage of the solidification can be considered as a pure conduction problem as was done by the "English" group.

[20] In this book, this equation because of its importance is referred to as Eckert–BarMeir's equation. If you have good experimental work, your name can be added to this equation.

3.5 Similarity applied to Die cavity

This section is useful for those who are dealing with research on die casting and or other casting process.

3.5.1 Governing equations

The filling of the mold cavity can be divided into two periods. In the first period (only fluid mechanics; minimum heat transfer/solidification) and the second period in which the solidification and dissipation occur. This discussion deals with how to conduct experiments in die casting[21]. It has to be stressed that the conditions down–stream have to be understood prior to the experiment with the die filling. The liquid metal velocity profile and flow pattern are still poorly understood at this stage. However, in this discussion we will assume that they are known or understood to same degree[22].

The governing equations are given in the preceding sections and now the boundary conditions will be discussed. The boundary condition at the solid interface for the gas/air and for the liquid metal are assumed to be "no–slip" condition which reads

$$u_g = v_g = w_g = u_{lm} = v_{lm} = w_{lm} = 0 \tag{3.57}$$

where the subscript g is used to indicate the gas phase. It is noteworthy to mention that this can be applied to the case where liquid metal is mixed with air/gas and both are touching the surface. At the interface between the liquid metal and gas/air, the pressure jump is expressed as

$$\frac{\sigma}{r_1 + r_2} \approx \Delta p \tag{3.58}$$

where r_1 and r_2 are the principal radii of the free surface curvature, and, σ, is the surface tension between the gas and the liquid metal. The surface geometry is determined by several factors which include the liquid movement[23] instabilities etc.

Now on the difficult parts, the velocity at gate has to be determined from the pQ^2 diagram or previous studies on the runner and shot sleeve. The difficulties arise due to the fact that we cannot assign a specific constant velocity and assume only liquid flow out. It has to be realized that due to the mixing processes in the shot sleeve and the runner (especially in a poor design process and runner system, now commonly used in the industry), some portions at the beginning of the process have a significant part which contains air/gas. There are several possibilities that the conditions can be prescribed. The first possibility is to describe the pressure variation at the entrance. The second possibility is to describe the velocity variation (as a function of time). The velocity is reduced during the filling of the cavity and is a function of the cavity geometry. The change in the velocity is sharp in the initial part of the filling due to the change

[21]Only minimal time and efforts was provided how to conduct experiments on the filling of the die. In the future, other zones and different processes will be discussed.

[22]Again the die casting process is a parabolic process.

[23]Note, the liquid surface cannot be straight, for unsteady state, because it results in no pressure gradient and therefore no movement.

from a free jet to an immersed jet. The pressure varies also at the entrance, however, the variations are more mild. Thus, it is a better possibility[24] to consider the pressure prescription. The simplest assumption is constant pressure

$$P = P_0 = \frac{1}{2}\rho {U_0}^2 \tag{3.59}$$

We also assume that the air/gas obeys the ideal gas model.

$$\rho_g = \frac{P}{RT} \tag{3.60}$$

where R is the air/gas constant and T is gas/air temperature. The previous assumption of negligible heat transfer must be inserted and further it has to be assumed that the process is polytropic[25]. The dimensionless gas density is defined as

$$\rho' = \frac{\rho}{\rho_0} = \left(\frac{P_0}{P}\right)^{\frac{1}{n}} \tag{3.61}$$

The subscript 0 denotes the atmospheric condition.

The air/gas flow rate out the cavity is assumed to behave according to the model in Chapter 9. Thus, the knowledge of the vent relative area and $\frac{4fL}{D}$ are important parameters. For cases where the vent is well designed (vent area is near the critical area or above the density, ρ_g can be determined as was done by [5]).

To study the controlling parameters, the equations are dimensionless–ed. The mass conservation for the liquid metal becomes

$$\frac{\partial \rho_{lm}}{\partial t'} + \frac{\partial \rho_{lm}\, {u'}_{lm}}{\partial x'} + \frac{\partial \rho_{lm}\, {v'}_{lm}}{\partial y'} + \frac{\partial \rho_{lm}\, {w'}_{lm}}{\partial z'} = 0 \tag{3.62}$$

where $x' = \frac{x}{\ell}$, $y' = y/\ell$, $z' = z/\ell$, $u' = u/U_0$, $v' = v/U_0$, $w' = w/U_0$ and the dimensionless time is defined as $t' = \frac{tU_0}{\ell}$, where $U_0 = \sqrt{2P_0/\rho}$.

Equation (3.62) can be similar under the assumption of constant density to read

$$\frac{\partial {u'}_{lm}}{\partial x'} + \frac{\partial {v'}_{lm}}{\partial y'} + \frac{\partial {w'}_{lm}}{\partial z'} = 0 \tag{3.63}$$

Please note that this simplification can be used for the gas phase. The momentum equation for the liquid metal in the x-coordinate assuming constant density and no body forces reads

$$\frac{\partial \rho_{lm} {u'}_{lm}}{\partial t'} + u' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial x'} + v' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial y'} + w' \frac{\partial \rho_{lm} {u'}_{lm}}{\partial z'} = \\ -\frac{\partial {p'}_{lm}}{\partial x'} + \frac{1}{Re}\left(\frac{\partial^2 u'_{lm}}{\partial {x'}^2} + \frac{\partial^2 v'}{\partial {y'_{lm}}^2} + \frac{\partial^2 w'}{\partial {z'_{lm}}^2}\right) \tag{3.64}$$

[24]At this only an intelligent guess is possible.

[25]There are several possibilities, this option is chosen only to obtain the main controlling parameters.

where $Re = U_0 \ell / \nu_{lm}$ and $p' = p/P_0$.

The gas phase continuity equation reads

$$\frac{\partial {\rho'}_g}{\partial t'} + \frac{\partial \rho'_g \, u'_g}{\partial x'} + \frac{\partial \rho'_g \, v'_g}{\partial y'} + \frac{\partial \rho'_g \, w'_g}{\partial z'} = 0 \tag{3.65}$$

The gas/air momentum equation[26] is transformed into

$$\frac{\partial \rho'_g {u'}_g}{\partial t'} + u' \frac{\partial \rho'_g {u'}_g}{\partial x'} + v' \frac{\partial \rho'_g {u'}_g}{\partial y'} + w' \frac{\partial \rho'_g {u'}_g}{\partial z'} = \\ - \frac{\partial {p'}_g}{\partial x'} + \underbrace{\frac{\nu_{lm}}{\nu_g} \frac{\rho_{g0}}{\rho_{lm}} \frac{1}{Re} \left(\frac{\partial^2 u'_g}{\partial {x'}^2} + \frac{\partial^2 v'_g}{\partial {y'}^2} + \frac{\partial^2 w'_g}{\partial {z'}^2} \right)}_{\sim 0} \tag{3.66}$$

Note that in this equation, additional terms were added, $(\nu_{lm}/\nu_g)(\rho_{g0}/\rho_{lm})$.

The "no-slip" conditions are converted to:

$$u'_g = v'_g = w'_g = u'_{lm} = v'_{lm} = w'_{lm} = 0 \tag{3.67}$$

The surface between the liquid metal and the air satisfy

$$p' \left(r'_1 + r'_2 \right) = \frac{1}{We} \tag{3.68}$$

where the p', r'_1, and r'_2 are defined as $r'_1 = r_1/\ell$ $r'_2 = r_2/\ell$

The solution to equations has the form of

$$\begin{aligned} u' &= f_u \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\ v' &= f_v \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\ w' &= f_w \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \\ p' &= f_p \left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n, \frac{\rho_g}{\rho_{lm}}, \frac{\nu_{lm}}{\nu_g} \right) \end{aligned} \tag{3.69}$$

If it will be found that equation (3.66) can be approximated[27] by

$$\frac{\partial {u'}_g}{\partial t'} + u' \frac{\partial {u'}_g}{\partial x'} + v' \frac{\partial {u'}_g}{\partial y'} + w' \frac{\partial {u'}_g}{\partial z'} \approx - \frac{\partial {p'}_g}{\partial x'} \tag{3.70}$$

---

[26] In writing this equation, it is assumed that viscosity of the air is independent of pressure and temperature.

[27] This topic is controversial in the area of two phase flow.

then the solution is reduced to

$$\begin{aligned}
u' &= f_u\left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n\right) \\
v' &= f_v\left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n\right) \\
w' &= f_w\left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n\right) \\
p' &= f_p\left(x', y', z', Re, We, \frac{A}{A_c}, \tfrac{4fL}{D}, n\right) \qquad (3.71)
\end{aligned}$$

At this stage, it is not known if it is the case and if it has to come out from the experiments. The density ratio can play a role because two phase flow characteristic is a major part of the filling process.

3.5.2 Design of Experiments

Under Construction [28].

3.6 *Summary of dimensionless numbers*

This section summarizes all the major dimensionless parameters and what effects they have on the die casting process.

Reynolds number

$$Re = \frac{\rho U^2/\ell}{\nu U/\ell^2} = \frac{\text{internal Forces}}{\text{viscous forces}}$$

Reynolds number represents the ratio of the momentum forces to the viscous forces. In die casting, Reynolds number plays a significant role which determines the flow pattern in the runner and the vent system. The discharge coefficient, C_D, is used in the pQ^2 diagram is determined largely by the Re number through the value of friction coefficient, f, inside the runner.

Eckert number

$$Ec = \frac{1/2\rho U^2}{1/2\rho c_p \Delta T} = \frac{\text{inertial energy}}{\text{thermal energy}}$$

Eckert number determines if the role of the momentum energy transferred to thermal energy is significant.

[28] See for time being Eckert's paper

Brinkman number

$$Br = \frac{\mu U^2/\ell^2}{k\Delta T/\ell^2} = \frac{\text{heat production by viscous dissipation}}{\text{heat transfer transport by conduction}}$$

Brinkman number is a measure of the importance of the viscous heating relative the conductive heat transfer. This number is important in cases where large velocity change occurs over short distances such as lubricant flow (perhaps, the flow in the gate). In die casting, this number has small values indicating that practically the viscous heating is not important.

Mach number

$$Ma = \frac{U}{\sqrt{\frac{\gamma \partial p}{\partial \rho}}}$$

For ideal gas (good assumption for the mixture of the gas leaving the cavity). It becomes

$$M \cong \frac{U}{\sqrt{\gamma RT}} = \frac{\text{characteristic velocity}}{\text{gas sound velocity}}$$

Mach number determines the characteristic of flow in the vent system where the air/gas velocity is reaching to the speed of sound. The air is chocked at the vent exit and in some cases other locations as well for vacuum venting. In atmospheric venting the flow is not chocked for large portion of the process. Moreover, the flow, in well design vent system, is not chocked. Yet the air velocity is large enough so that the Mach number has to be taken into account for reasonable calculation of the C_D.

Ozer number

$$Oz = \frac{\frac{{C_D}^2 P_{max}}{\rho}}{\left(\frac{Q_{max}}{A_3}\right)^2} = \left(\frac{A_3}{Q_{max}}\right)^2 {C_D}^2 \frac{P_{max}}{\rho} = \frac{\text{effective static pressure energy}}{\text{average kinematic energy}}$$

One of the most important number in the pQ^2 diagram calculation is Ozer number. This number represents how good the runner is designed.

Froude number

$$Fr = \frac{\rho U^2/\ell}{\rho g} = \frac{\text{inertial forces}}{\text{gravity forces}}$$

Fr number represent the ratio of the gravity forces to the momentum forces. It is very important in determining the critical slow plunger velocity. This number is determined by the height of the liquid metal in the shot sleeve. The Froude number does not play a significant role in the filling of the cavity.

Capillary number

$$Ca = \frac{\rho U^2/\ell}{\rho g} = \frac{\text{inertial forces}}{\text{gravity forces}}$$

capillary number (Ca) determine when the flow during the filling of the cavity is atomized or is continuous flow (for relatively low Re number).

Weber number

$$We = \frac{1/2\rho U^2}{1/2\sigma/\ell} = \frac{\text{inertial forces}}{\text{surface forces}}$$

We number is the other parameter that govern the flow pattern in the die. The flow in die casting is atomized and, therefore, We with combinations of the gate design also determine the drops sizes and distribution.

Critical vent area

$$A_c = \frac{V(0)}{c t_{max} m_{max}}$$

The critical area is the area for which the air/gas is well vented.

3.7 Summary

The dimensional analysis demonstrates that the fluid mechanics process, such as the filling of the cavity with liquid metal and evacuation/extraction of the air from the mold, can be dealt with when heat transfer is neglected. This provides an excellent opportunity for simple models to predict many parameters in the die casting process. It is recommended for interested readers to read Eckert's book "Analysis of Heat and Mass transfer" to have better and more general understanding of this topic.

3.8 Questions

Under construction