Note:

CHAPTER 9: GRAVITY

Version: 0.4.8.8 January 1, 2012

This chapter is part of the textbook:

"Fundamentals of Compressible Flow"

You can download the whole book if you like from: *www.potto.org.*

Please be aware that this book is updated frequently — every three weeks or so.

GENICK BAR-MEIR, PH.D.
CHICAGO, ILLINOIS
JANUARY 1, 2012

THE LIST OF THE AVAILABLE BOOKS IN POTTO PROJECT

Project Name	Progress	Remarks	Version	Availability for Public Download
Compressible Flow	beta		0.4.8.6	✔
Gas Dynamics Tables	final	World biggest	1.2	✔
Die Casting	alpha		0.1.2	✔
Dynamics	NSY		0.0.0	✖
Fluid Mechanics	beta		0.2.9	✔
Heat Transfer	NSY	Based on Eckert	0.0.0	✖
Mechanics	NSY		0.0.0	✖
Open Channel Flow	NSY		0.0.0	✖
Statics	early alpha	first chapter	0.0.1	✖
Strength of Material	NSY		0.0.0	✖
Thermodynamics	early alpha		0.0.01	✖
Two/Multi phases flow	NSY	Tel-Aviv's notes	0.0.0	✖

NSY = Not Started Yet

CHAPTER 8

Nozzle Flow With External Forces

This chapter is under heavy construction. Please ignore. If you want to contribute and add any results of experiments, to this chapter, please do so. You can help especially if you have photos showing these effects.

In the previous chapters a simple model describing the flow in nozzle was explained. In cases where more refined calculations have to carried the gravity or other forces have to be taken into account. Flow in a vertical or horizontal nozzle are different because the gravity. The simplified models that suggests them–self are: friction and adiabatic, isothermal, seem the most applicable. These models can served as limiting cases for more realistic flow.

The effects of the gravity of the nozzle flow in two models isentropic and isothermal is analyzed here. The isothermal nozzle model is suitable in cases where the flow is relatively slow (small Eckert numbers) while as the isentropic model is more suitable for large Eckert numbers.

The two models produces slightly different equations. The equations results in slightly different conditions for the chocking and different chocking speed. Moreover, the working equations are also different and this author isn't aware of material in the literature which provides any working table for the gravity effect.

8.1 Isentropic Nozzle ($Q = 0$)

The energy equation for isentropic nozzle provides

$$dh + UdU = \overbrace{f(x)dx}^{\text{external work or potential difference, i.e. } z \times g} \tag{8.1}$$

Utilizing equation (5.27) when $ds = 0$ leads to

$$\frac{dP}{\rho} + UdU = f(x')dx' \tag{8.2}$$

For the isentropic process $dP = const \times k\rho^{k-1}d\rho$ when the $const = P/\rho^k$ at any point of the flow. The equation (8.2) becomes

$$\begin{aligned} \overbrace{\overbrace{\frac{P}{\rho^k}}^{any\ point} \quad k\frac{\rho^k}{\rho}d\rho}^{dP}\frac{1}{\rho} + UdU = k\overbrace{\frac{P}{\rho}}^{RT}\frac{d\rho}{\rho}UdU &= f(x')dx' \\ \frac{kRTd\rho}{\rho} + UdU = \frac{c^2}{\rho}d\rho + UdU &= f(x')dx' \end{aligned} \tag{8.3}$$

The continuity equation as developed earlier (mass conservation equation isn't effected by the gravity)

$$-\frac{d\rho}{\rho} = \frac{dA}{A} + \frac{dU}{U} = 0 \tag{8.4}$$

Substituting $d\rho/\rho$ from equation 8.3, into equation (8.2) moving $d\rho$ to the right hand side, and diving by dx' yields

$$U\frac{dU}{dx'} = c^2\left[\frac{1}{U}\frac{dU}{dx'} + \frac{1}{A}\frac{dA}{dx'}\right] + f(x') \tag{8.5}$$

Rearranging equation (8.5) yields

$$\frac{dU}{dx'} = \left[M^2\frac{dU}{dx'} + \frac{c^2}{AU}\frac{dA}{dx'}\right] + \frac{f(x')}{U} \tag{8.6}$$

And further rearranging yields

$$\left(1 - M^2\right)\frac{dU}{dx'} = \frac{c^2}{AU}\frac{dA}{dx'} + \frac{f(x')}{U} \tag{8.7}$$

Equation (8.7) can be rearranged as

$$\frac{dU}{dx'} = \frac{1}{(1-M^2)}\left[\frac{c^2}{AU}\frac{dA}{dx'} + \frac{f(x')}{U}\right] \tag{8.8}$$

Equation (8.8) dimensionless form by utilizing $x = x'/\ell$ and ℓ is the nozzle length

$$\frac{dM}{dx} = \frac{1}{(1-M^2)}\left[\frac{1}{AM}\frac{dA}{dx} + \frac{\ell f(x)}{c\,\underbrace{cM}_{U}}\right] \tag{8.9}$$

And the final form of equation (8.9) is

$$\frac{d\left(M^2\right)}{dx} = \frac{2}{(1-M^2)}\left[\frac{1}{A}\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] \tag{8.10}$$

The term $\frac{\ell f(x)}{c^2}$ is considered to be very small ($0.1 \times 10/100000 < 0.1\%$) for "standard" situations. The dimensionless number, $\frac{\ell f(x)}{c^2}$ sometimes referred as Ozer number determines whether gravity should be considered in the calculations. Nevertheless, one should be aware of value of Ozer number for large magnetic fields (astronomy) and low temperature, In such cases, the gravity effect can be considerable.

As it was shown before the transition must occur when $M = 1$. Consequently, two zones must be treated separately. First, here the Mach number is discussed and not the pressure as in the previous chapter. For $M < 1$ (the subsonic branch) the term $\frac{2}{(1-M^2)}$ is positive and the treads determined by gravity and the area function.

$$\left[\frac{1}{A}\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] > 0 \Longrightarrow d(M^2) > 0$$

or conversely,

$$\left[\frac{1}{A}\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] < 0 \Longrightarrow d(M^2) < 0$$

For the case of $M > 1$ (the supersonic branch) the term $\frac{2}{(1-M^2)}$ is negative and therefore

$$\left[\frac{1}{A}\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] > 0 \Longrightarrow d(M^2) < 0$$

For the border case $M = 1$, the denominator $1 - M^2 = 0$, is zero either $d(M^2) = \infty$ or

$$\left[\frac{1}{A}\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] = 0.$$

And the dM is indeterminate. As it was shown in chapter (5) the flow is chocked ($M = 1$) only when

$$\left[\frac{dA}{dx} + \frac{\ell f(x)}{c^2}\right] = 0. \tag{8.11}$$

It should be noticed that when $f(x)$ is zero, e.g. horizontal flow, the equation (8.11) reduced into $\frac{dA}{dx} = 0$ that was developed previously.

The ability to manipulate the location provides a mean to increase/decrease the flow rate. Yet this ability since Ozer number is relatively very small.

This condition means that the critical point can occurs in several locations that satisfies equation (8.11). Further, the critical point, sonic point is $\frac{dA}{Ax} \neq 0$ If $f(x)$ is a positive function, the critical point happen at converging part of the nozzle (before the throat) and if $f(x)$ is a negative function the critical point is diverging part of the throat. For example consider the gravity, $f(x) = -g$ a flow in a nozzle vertically the critical point will be above the throat.

8.2 Isothermal Nozzle $(T = constant)$