

Note:

CHAPTER 12: RAYLEIGH

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This chapter is part of the textbook:

**“Fundamentals of Compressible
Flow”**

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NSY = Not Started Yet

CHAPTER 11

Rayleigh Flow

Rayleigh flow is a model describing a frictionless flow with heat transfer through a pipe of constant cross-sectional area. In practice, Rayleigh flow isn't a really good model for the real situation. Yet, Rayleigh flow is a practical and useful concept in obtaining trends and limits such as the density and pressure change due to external cooling or heating. As opposed to the two previous models, the heat transfer can be in two directions, not like the friction (there is no negative friction). This fact creates a situation different as compared to the previous two models. This model can be applied to cases where the heat transfer is significant and the friction can be ignored.

11.1 Introduction

The third simple model for one-dimensional flow with constant heat transfer for frictionless flow. This flow is referred to in the literature as Rayleigh Flow (see historical notes). This flow is another extreme case in which the friction effect is neglected because their relative effect is much smaller than the heat transfer effect. While the isothermal flow model has heat transfer and friction, the main assumption was that relative length is enables significant heat transfer to occur between the surroundings and tube. In contrast, the heat transfer in Rayleigh flow occurs between unknown temperature and the tube and the heat flux is maintained constant. As before, a simple model is built around the assumption of constant properties (poorer prediction to cases where chemical reactions take place).

This model is used to roughly predict the conditions which occur mostly in sit-

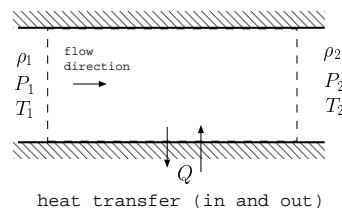


Fig. -11.1. The control volume of Rayleigh Flow

uations involving chemical reaction. In analysis of the flow, one has to be aware that properties do change significantly for a large range of temperatures. Yet, for smaller range of temperatures and lengths the calculations are more accurate. Nevertheless, the main characteristic of the flow such as a choking condition etc. is encapsulated in this model.

The basic physics of the flow revolves around the fact that the gas is highly compressible. The density changes through the heat transfer (temperature change). Contrary to Fanno flow in which the resistance always oppose the flow direction, Rayleigh flow, also, the cooling can be applied. The flow velocity acceleration change the direction when the cooling is applied.

11.2 Governing Equation

The energy balance on the control volume reads

$$Q = C_p (T_{02} - T_{01}) \quad (11.1)$$

the momentum balance reads

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1) \quad (11.2)$$

The mass conservation reads

$$\rho_1 U_1 A = \rho_2 U_2 A = \dot{m} \quad (11.3)$$

Equation of state

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad (11.4)$$

There are four equations with four unknowns, if the upstream conditions are known (or downstream conditions are known). Thus, a solution can be obtained. One can notice that equations (11.2), (11.3) and (11.4) are similar to the equations that were solved for the shock wave.

$$\frac{P_2}{P_1} = \frac{1 + kM_1^2}{1 + kM_2^2} \quad (11.5)$$

The equation of state (11.4) can further assist in obtaining the temperature ratio as

$$\frac{T_2}{T_1} = \frac{P_2 \rho_1}{P_1 \rho_2} \quad (11.6)$$

The density ratio can be expressed in terms of mass conservation as

$$\frac{\rho_1}{\rho_2} = \frac{U_2}{U_1} = \frac{\frac{U_2}{\sqrt{kRT_2}} \sqrt{kRT_2}}{\frac{U_1}{\sqrt{kRT_1}} \sqrt{kRT_1}} = \frac{M_2}{M_1} \sqrt{\frac{T_2}{T_1}} \quad (11.7)$$

or

$$\frac{\rho_1}{\rho_2} = \frac{U_2}{U_1} = \frac{M_2}{M_1} \sqrt{\frac{T_2}{T_1}} \tag{11.8}$$

or Substituting equations (11.5) and (11.8) into equation (11.6) yields

$$\frac{T_2}{T_1} = \frac{1 + kM_1^2}{1 + kM_2^2} \frac{M_2}{M_1} \sqrt{\frac{T_2}{T_1}} \tag{11.9}$$

Transferring the temperature ratio to the left hand side and squaring the results gives

$$\frac{T_2}{T_1} = \left[\frac{1 + kM_1^2}{1 + kM_2^2} \right]^2 \left(\frac{M_2}{M_1} \right)^2 \tag{11.10}$$

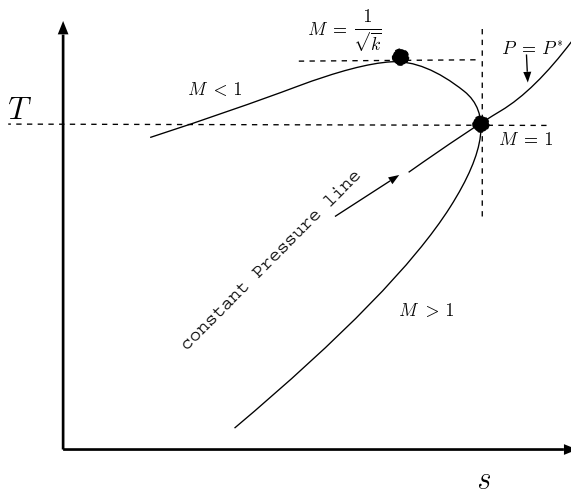


Fig. -11.2. The temperature entropy diagram for Rayleigh line

The Rayleigh line exhibits two possible maximums one for $dT/ds = 0$ and for $ds/dT = 0$. The second maximum can be expressed as $dT/ds = \infty$. The second law is used to find the expression for the derivative.

$$\frac{s_1 - s_2}{C_p} = \ln \frac{T_2}{T_1} - \frac{k - 1}{k} \ln \frac{P_2}{P_1} \tag{11.11}$$

$$\frac{s_1 - s_2}{C_p} = 2 \ln \left[\left(\frac{1 + kM_1^2}{1 + kM_2^2} \right) \frac{M_2}{M_1} \right] + \frac{k - 1}{k} \ln \left[\frac{1 + kM_2^2}{1 + kM_1^2} \right] \tag{11.12}$$

Let the initial condition M_1 , and s_1 be constant and the variable parameters are M_2 , and s_2 . A derivative of equation (11.12) results in

$$\frac{1}{C_p} \frac{ds}{dM} = \frac{2(1 - M^2)}{M(1 + kM^2)} \quad (11.13)$$

Taking the derivative of equation (11.13) and letting the variable parameters be T_2 , and M_2 results in

$$\frac{dT}{dM} = \text{constant} \times \frac{1 - kM^2}{(1 + kM^2)^3} \quad (11.14)$$

Combining equations (11.13) and (11.14) by eliminating dM results in

$$\frac{dT}{ds} = \text{constant} \times \frac{M(1 - kM^2)}{(1 - M^2)(1 + kM^2)^2} \quad (11.15)$$

On T-s diagram a family of curves can be drawn for a given constant. Yet for every curve, several observations can be generalized. The derivative is equal to zero when $1 - kM^2 = 0$ or $M = 1/\sqrt{k}$ or when $M \rightarrow 0$. The derivative is equal to infinity, $dT/ds = \infty$ when $M = 1$. From thermodynamics, increase of heating results in increase of entropy. And cooling results in reduction of entropy. Hence, when cooling is applied to a tube the velocity decreases and when heating is applied the velocity increases. At peculiar point of $M = 1/\sqrt{k}$ when additional heat is applied the temperature decreases. The derivative is negative, $dT/ds < 0$, yet note this point is not the choking point. The choking occurs only when $M = 1$ because it violates the second law. The transition to supersonic flow occurs when the area changes, somewhat similarly to Fanno flow. Yet, choking can be explained by the fact that increase of energy must be accompanied by increase of entropy. But the entropy of supersonic flow is lower (see Figure (11.2)) and therefore it is not possible (the maximum entropy at $M = 1$).

It is convenient to refer to the value of $M = 1$. These values are referred to as the "star"¹ values. The equation (11.5) can be written between choking point and any point on the curve.

$$\frac{P^*}{P_1} = \frac{1 + kM_1^2}{1 + k} \quad (11.16)$$

The temperature ratio is

$$\frac{T^*}{T_1} = \frac{1}{M^2} \left(\frac{1 + kM_1^2}{1 + k} \right)^2 \quad (11.17)$$

¹The star is an asterisk.

$$\frac{\rho_1}{\rho^*} = \frac{U^*}{U_1} = \frac{\frac{U^*}{\sqrt{kRT^*}} \sqrt{kRT^*}}{\frac{U_1}{\sqrt{kRT_1}} \sqrt{kRT_1}} \quad (11.18)$$

or

$$\frac{\rho_1}{\rho^*} = \frac{U^*}{U_1} = \frac{1}{M_1} \sqrt{\frac{T^*}{T_1}} \quad (11.19)$$

$$\frac{T_{01}}{T_0^*} = \frac{T_1 \left(1 + \frac{k-1}{2} M_1^2\right)}{T^* \left(\frac{1+k}{2}\right)} \quad (11.20)$$

or explicitly

$$\frac{T_{01}}{T_0^*} = \frac{2(1+k)M_1^2}{(1+kM_1^2)^2} \left(1 + \frac{k-1}{2} M_1^2\right) \quad (11.21)$$

The stagnation pressure ratio reads

$$\frac{P_{01}}{P_0^*} = \frac{P_1 \left(1 + \frac{k-1}{2} M_1^2\right)}{P^* \left(\frac{1+k}{2}\right)} \quad (11.22)$$

or explicitly

$$\frac{P_{01}}{P_0^*} = \left(\frac{1+k}{1+kM_1^2}\right) \left(\frac{1+kM_1^2}{\frac{(1+k)}{2}}\right)^{\frac{k}{k-1}} \quad (11.23)$$

11.3 Rayleigh Flow Tables

The “star” values are tabulated in Table (11.1). Several observations can be made in regards to the stagnation temperature.

Table -11.1. Rayleigh Flow $k=1.4$

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.03	0.00517	0.00431	2.397	1.267	0.00216
0.04	0.00917	0.00765	2.395	1.266	0.00383

Table -11.1. Rayleigh Flow $k=1.4$ (continue)

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.05	0.014300	0.011922	2.392	1.266	0.00598
0.06	0.020529	0.017119	2.388	1.265	0.00860
0.07	0.027841	0.023223	2.384	1.264	0.011680
0.08	0.036212	0.030215	2.379	1.262	0.015224
0.09	0.045616	0.038075	2.373	1.261	0.019222
0.10	0.056020	0.046777	2.367	1.259	0.023669
0.20	0.20661	0.17355	2.273	1.235	0.090909
0.25	0.30440	0.25684	2.207	1.218	0.13793
0.30	0.40887	0.34686	2.131	1.199	0.19183
0.35	0.51413	0.43894	2.049	1.178	0.25096
0.40	0.61515	0.52903	1.961	1.157	0.31373
0.45	0.70804	0.61393	1.870	1.135	0.37865
0.50	0.79012	0.69136	1.778	1.114	0.44444
0.55	0.85987	0.75991	1.686	1.094	0.51001
0.60	0.91670	0.81892	1.596	1.075	0.57447
0.65	0.96081	0.86833	1.508	1.058	0.63713
0.70	0.99290	0.90850	1.423	1.043	0.69751
0.75	1.014	0.94009	1.343	1.030	0.75524
0.80	1.025	0.96395	1.266	1.019	0.81013
0.85	1.029	0.98097	1.193	1.011	0.86204
0.90	1.025	0.99207	1.125	1.005	0.91097
0.95	1.015	0.99814	1.060	1.001	0.95693
1.0	1.00	1.00	1.00	1.00	1.000
1.1	0.96031	0.99392	0.89087	1.005	1.078
1.2	0.91185	0.97872	0.79576	1.019	1.146
1.3	0.85917	0.95798	0.71301	1.044	1.205
1.4	0.80539	0.93425	0.64103	1.078	1.256
1.5	0.75250	0.90928	0.57831	1.122	1.301
1.6	0.70174	0.88419	0.52356	1.176	1.340
1.7	0.65377	0.85971	0.47562	1.240	1.375
1.8	0.60894	0.83628	0.43353	1.316	1.405
1.9	0.56734	0.81414	0.39643	1.403	1.431
2.0	0.52893	0.79339	0.36364	1.503	1.455
2.1	0.49356	0.77406	0.33454	1.616	1.475
2.2	0.46106	0.75613	0.30864	1.743	1.494
2.3	0.43122	0.73954	0.28551	1.886	1.510
2.4	0.40384	0.72421	0.26478	2.045	1.525
2.5	0.37870	0.71006	0.24615	2.222	1.538
2.6	0.35561	0.69700	0.22936	2.418	1.550

Table -11.1. Rayleigh Flow $k=1.4$ (continue)

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
2.7	0.33439	0.68494	0.21417	2.634	1.561
2.8	0.31486	0.67380	0.20040	2.873	1.571
2.9	0.29687	0.66350	0.18788	3.136	1.580
3.0	0.28028	0.65398	0.17647	3.424	1.588
3.5	0.21419	0.61580	0.13223	5.328	1.620
4.0	0.16831	0.58909	0.10256	8.227	1.641
4.5	0.13540	0.56982	0.081772	12.50	1.656
5.0	0.11111	0.55556	0.066667	18.63	1.667
5.5	0.092719	0.54473	0.055363	27.21	1.675
6.0	0.078487	0.53633	0.046693	38.95	1.681
6.5	0.067263	0.52970	0.039900	54.68	1.686
7.0	0.058264	0.52438	0.034483	75.41	1.690
7.5	0.050943	0.52004	0.030094	1.0E+2	1.693
8.0	0.044910	0.51647	0.026490	1.4E+2	1.695
8.5	0.039883	0.51349	0.023495	1.8E+2	1.698
9.0	0.035650	0.51098	0.020979	2.3E+2	1.699
9.5	0.032053	0.50885	0.018846	3.0E+2	1.701
10.0	0.028972	0.50702	0.017021	3.8E+2	1.702
20.0	0.00732	0.49415	0.00428	1.1E+4	1.711
25.0	0.00469	0.49259	0.00274	3.2E+4	1.712
30.0	0.00326	0.49174	0.00190	8.0E+4	1.713
35.0	0.00240	0.49122	0.00140	1.7E+5	1.713
40.0	0.00184	0.49089	0.00107	3.4E+5	1.714
45.0	0.00145	0.49066	0.000846	6.0E+5	1.714
50.0	0.00117	0.49050	0.000686	1.0E+6	1.714
55.0	0.000971	0.49037	0.000567	1.6E+6	1.714
60.0	0.000816	0.49028	0.000476	2.5E+6	1.714
65.0	0.000695	0.49021	0.000406	3.8E+6	1.714
70.0	0.000600	0.49015	0.000350	5.5E+6	1.714

The data is presented in Figure (11.3).

11.4 Examples For Rayleigh Flow

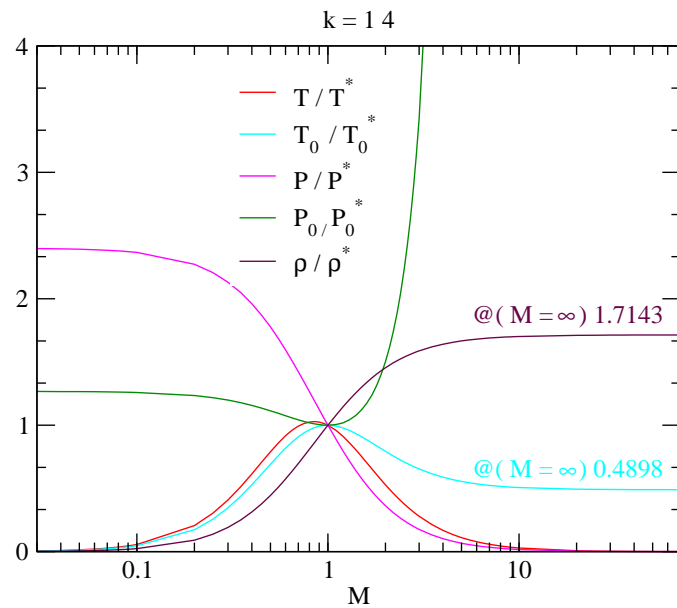
Illustrative example

The typical questions that are raised in Rayleigh Flow are related to the maximum heat that can be transferred to gas (reaction heat) and to the flow rate.

Example 11.1:

Air enters a pipe with pressure of 3[bar] and temperature of 27°C at Mach number of

Rayleigh Flow



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Fig. -11.3. The basic functions of Rayleigh Flow ($k=1.4$)

$M = 0.25$. Due to internal combustion heat was released and the exit temperature was found to be 127°C . Calculate the exit Mach number, the exit pressure, the total exit pressure, and heat released and transferred to the air. At what amount of energy the exit temperature will start to decrease? Assume $C_P = 1.004 \left[\frac{\text{kJ}}{\text{kg}^\circ\text{C}} \right]$

SOLUTION

The entrance Mach number and the exit temperature are given and from Table (11.1) or from the program the initial ratio can be calculated. From the initial values the ratio at the exit can be computed as the following.

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.25000	0.30440	0.25684	2.2069	1.2177	0.13793

and

$$\frac{T_2}{T^*} = \frac{T_1}{T^*} \frac{T_2}{T_1} = 0.304 \times \frac{400}{300} = 0.4053$$

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.29831	0.40530	0.34376	2.1341	1.1992	0.18991

The exit Mach number is known, the exit pressure can be calculated as

$$P_2 = P_1 \frac{P^*}{P_1} \frac{P_2}{P^*} = 3 \times \frac{1}{2.2069} \times 2.1341 = 2.901[Bar]$$

For the entrance the stagnation values are

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
0.25000	0.98765	0.96942	2.4027	0.95745	2.3005	1.0424

The total exit pressure, P_{0_2} can be calculated as the following:

$$P_{0_2} = P_1 \overbrace{\frac{P_{0_1}}{P_1}}^{isentropic} \frac{P_0^*}{P_{0_1}} \frac{P_{0_2}}{P_0^*} = 3 \times \frac{1}{0.95745} \times \frac{1}{1.2177} \times 1.1992 = 3.08572[Bar]$$

The heat released (heat transferred) can be calculated from obtaining the stagnation temperature from both sides. The stagnation temperature at the entrance, T_{0_1}

$$T_{0_1} = T_1 \overbrace{\frac{T_{0_1}}{T_1}}^{isentropic} = 300/0.98765 = 303.75[K]$$

The isentropic conditions at the exit are

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
0.29831	0.98251	0.95686	2.0454	0.94012	1.9229	0.90103

The exit stagnation temperature is

$$T_{0_2} = T_2 \overbrace{\frac{T_{0_2}}{T_2}}^{isentropic} = 400/0.98765 = 407.12[K]$$

The heat released becomes

$$\frac{Q}{\dot{m}} = C_p (T_{0_2} - T_{0_1}) 1 \times 1.004 \times (407.12 - 303.75) = 103.78 \left[\frac{kJ}{sec \cdot kg^\circ C} \right]$$

The maximum temperature occurs at the point where the Mach number reaches $1/\sqrt{k}$ and at this point the Rayleigh relationship are:

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.84515	1.0286	0.97959	1.2000	1.0116	0.85714

The maximum heat before the temperature can be calculated as following:

$$T_{max} = T_1 \frac{T^*}{T_1} \frac{T_{max}}{T^*} \frac{300}{0.3044} \times 1.0286 = 1013.7[K]$$

The isentropic relationship at the maximum are

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
0.84515	0.87500	0.71618	1.0221	0.62666	0.64051	0.53376

The stagnation temperature for this point is

$$T_{0_{max}} = T_{max} * \frac{T_{0_{max}}}{T_{max}} = \frac{1013.7}{0.875} = 1158.51[K]$$

The maximum heat can be calculated as

$$\frac{Q}{\dot{m}} = C_p (T_{0_{max}} - T_{0_1}) = 1 \times 1.004 \times (1158.51 - 303.75) = 858.18 \left[\frac{kJ}{kgsecK} \right]$$

Note that this point isn't the choking point.

End Solution

Example 11.2:

Heat is added to the air until the flow is choked in amount of 600 [kJ/kg]. The exit temperature is 1000 [K]. Calculate the entrance temperature and the entrance Mach number.

SOLUTION

The solution involves finding the stagnation temperature at the exit and subtracting the heat (heat equation) to obtain the entrance stagnation temperature. From the Table (11.1) or from the Potto-GDC the following ratios can be obtained.

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
1.0000	0.83333	0.63394	1.0000	0.52828	0.52828	0.52828

The stagnation temperature

$$T_{0_2} = T_2 \frac{T_{0_2}}{T_2} = \frac{1000}{0.83333} = 1200.0[K]$$

The entrance temperature is

$$\frac{T_{01}}{T_{02}} = 1 - \frac{Q/\dot{m}}{T_{02}C_P} = 1200 - \frac{600}{1200 \times 1.004} \cong 0.5016$$

It must be noted that $T_{02} = T_0^*$. Therefore with $\frac{T_{01}}{T_0^*} = 0.5016$ either by using Table (11.1) or by Potto-GDC the following is obtained

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.34398	0.50160	0.42789	2.0589	1.1805	0.24362

Thus, entrance Mach number is 0.38454 and the entrance temperature can be calculated as following

$$T_1 = T^* \frac{T_1}{T^*} = 1000 \times 0.58463 = 584.6[K]$$

End Solution

The difference between the supersonic branch to subsonic branch

Example 11.3:

Air with Mach 3 enters a frictionless duct with heating. What is the maximum heat that can be added so that there is no subsonic flow? If a shock occurs immediately at the entrance, what is the maximum heat that can be added?

SOLUTION

To achieve maximum heat transfer the exit Mach number has to be one, $M_2 = 1$.

$$\frac{Q}{\dot{m}} = C_p (T_{02} - T_{01}) = C_p T_0^* \left(1 - \frac{T_{01}}{T_0^*} \right)$$

The table for $M = 3$ as follows

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
3.0000	0.28028	0.65398	0.17647	3.4245	1.5882

The higher the entrance stagnation temperature the larger the heat amount that can be absorbed by the flow. In subsonic branch the Mach number after the shock is

M_x	M_y	$\frac{T_y}{T_x}$	$\frac{\rho_y}{\rho_x}$	$\frac{P_y}{P_x}$	$\frac{P_{0y}}{P_{0x}}$
3.0000	0.47519	2.6790	3.8571	10.3333	0.32834

With Mach number of $M = 0.47519$ the maximum heat transfer requires information for Rayleigh flow as the following

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.33138	0.47519	0.40469	2.0802	1.1857	0.22844

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.47519	0.75086	0.65398	1.8235	1.1244	0.41176

It also must be noticed that stagnation temperature remains constant across shock wave.

$$\frac{\frac{Q}{\dot{m}} \Big|_{subsonic}}{\frac{Q}{\dot{m}} \Big|_{supersonic}} = \frac{\left(1 - \frac{T_{01}}{T_0^*}\right)_{subsonic}}{\left(1 - \frac{T_{01}}{T_0^*}\right)_{supersonic}} = \frac{1 - 0.65398}{1 - 0.65398} = 1$$

It is not surprising for the shock wave to be found in the Rayleigh flow.

End Solution

Example 11.4:

One of the reason that Rayleigh flow model was invented is to be analyzed the flow in a combustion chamber. Consider a flow of air in conduct with a fuel injected into the flow as shown in Figure 11.4. Calculate what the maximum fuel-air ratio. Calculate the exit condition for half the fuel-air ratio.

Assume that the mixture properties are of air. Assume that the combustion heat is 25,000[KJ/kg fuel] for the average temperature range for this mixture. Neglect the fuel mass addition and assume that all the fuel is burned (neglect the complications of the increase of the entropy if accrue).

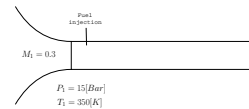


Fig. -11.4. Schematic of the combustion chamber.

SOLUTION

Under these assumptions the maximum fuel air ratio is obtained when the flow is choked. The entranced condition can be obtained using Potto-GDC as following

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.30000	0.40887	0.34686	2.1314	1.1985	0.19183

The choking condition are obtained using also by Potto-GDC as

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

And the isentropic relationships for Mach 0.3 are

M	$\frac{T}{T_0}$	$\frac{\rho}{\rho_0}$	$\frac{A}{A^*}$	$\frac{P}{P_0}$	$\frac{A \times P}{A^* \times P_0}$	$\frac{F}{F^*}$
0.30000	0.98232	0.95638	2.0351	0.93947	1.9119	0.89699

The maximum fuel-air can be obtained by finding the heat per unit mass.

$$\frac{\dot{Q}}{\dot{m}} = \frac{Q}{m} = C_p (T_{02} - T_{01}) = C_p T_1 \left(1 - \frac{T_{01}}{T^*} \right)$$

$$\frac{\dot{Q}}{\dot{m}} = 1.04 \times 350 / 0.98232 \times (1 - 0.34686) \sim 242.022 [kJ/kg]$$

The fuel-air mass ratio has to be

$$\frac{m_{fuel}}{m_{air}} = \frac{\text{needed heat}}{\text{combustion heat}} = \frac{242.022}{25,000} \sim 0.0097 [kg \text{ fuel} / kg \text{ air}]$$

If only half of the fuel is supplied then the exit temperature is

$$T_{02} = \frac{Q}{m C_p} + T_{01} = \frac{0.5 \times 242.022}{1.04} + 350 / 0.98232 \sim 472.656 [K]$$

The exit Mach number can be determined from the exit stagnation temperature as following:

$$\frac{T_2}{T^*} = \frac{T_{01}}{T_0^*} \frac{T_{02}}{T_{01}}$$

The last temperature ratio can be calculated from the value of the temperatures

$$\frac{T_2}{T^*} = 0.34686 \times \frac{472.656}{350 / 0.98232} \sim 0.47685$$

The Mach number can be obtained from a Rayleigh table or using Potto-GDC

M	$\frac{T}{T^*}$	$\frac{T_0}{T_0^*}$	$\frac{P}{P^*}$	$\frac{P_0}{P_0^*}$	$\frac{\rho^*}{\rho}$
0.33217	0.47685	0.40614	2.0789	1.1854	0.22938

It should be noted that this example is only to demonstrate how to carry the calculations.

