

# Note:

CHAPTER 5: SOUND

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This chapter is part of the textbook:

**“Fundamentals of Compressible  
Flow”**

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Two/Multi phases flow	NSY	Tel-Aviv's notes	0.0.0	✗

NSY = Not Started Yet

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# CHAPTER 4

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## Speed of Sound

### 4.1 Motivation

In traditional compressible flow classes there is very little discussion about the speed of sound outside the ideal gas. The author thinks that this approach has many shortcomings. In a recent consultation an engineer<sup>1</sup> design a industrial system that contains converting diverging nozzle with filter to remove small particles from air. The engineer was well aware of the calculation of the nozzle. Thus, the engineer was able to predict that was a choking point. Yet, the engineer was not ware of the effect of particles on the speed of sound. Hence, the actual flow rate was only half of his prediction. As it will shown in this chapter, the particles can, in some situations, reduces the speed of sound by almost as half. With the “new” knowledge from the consultation the calculations were within the range of acceptable results.

The above situation is not unique in the industry. It should be expected that engineers know how to manage this situation of non pure substances (like clean air). The fact that the engineer knows about the choking is great but it is not enough for today's sophisticated industry<sup>2</sup>. In this chapter an introductory discussion is given about different situations which can appear the industry in regards to speed of sound.

### 4.2 Introduction

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<sup>1</sup>Aerospace engineer, alumni of University of Minnesota, Aerospace Department.

<sup>2</sup>Pardon, but a joke is must in this situation. A cat is pursuing a mouse and the mouse escape and hide in the hole. Suddenly, the mouse hear a barking dog and a cat yelling. The mouse go out to investigate, and cat caught the mouse. The mouse asked the cat I thought I heard a dog. The cat reply, yes you did. My teacher was right, one language is not enough today.

The people had recognized for several hundred years that sound is a variation of pressure. The ears sense the variations by frequency and magnitude which are transferred to the brain which translates to voice. Thus, it raises the question: what is the speed of the small disturbance travel in a "quiet" medium. This velocity is referred to as the speed of sound.

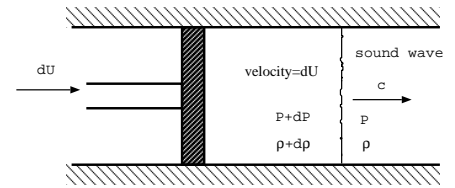


Fig. -4.1. A very slow moving piston in a still gas

To answer this question consider a piston moving from the left to the right at a relatively small velocity (see Figure 4.1). The information that the piston is moving passes through a single "pressure pulse." It is assumed that if the velocity of the piston is infinitesimally small, the pulse will be infinitesimally small. Thus, the pressure and density can be assumed to be continuous.

In the control volume it is convenient to look at a control volume which is attached to a pressure pulse. Applying the mass balance yields

$$\rho c = (\rho + d\rho)(c - dU) \quad (4.1)$$

or when the higher term  $dU d\rho$  is neglected yields

$$\rho dU = cd\rho \implies dU = \frac{cd\rho}{\rho} \quad (4.2)$$

From the energy equation (Bernoulli's equation), assuming isentropic flow and neglecting the gravity results

$$\frac{(c - dU)^2}{2} - \frac{c^2}{2} + \frac{dP}{\rho} = 0 \quad (4.3)$$

neglecting second term ( $dU^2$ ) yield

$$-cdU + \frac{dP}{\rho} = 0 \quad (4.4)$$

Substituting the expression for  $dU$  from equation (4.2) into equation (4.4) yields

$$c^2 \left( \frac{d\rho}{\rho} \right) = \frac{dP}{\rho} \implies c^2 = \frac{dP}{d\rho} \quad (4.5)$$

An expression is needed to represent the right hand side of equation (4.5). For an ideal gas,  $P$  is a function of two independent variables. Here, it is considered that

$P = P(\rho, s)$  where  $s$  is the entropy. The full differential of the pressure can be expressed as follows:

$$dP = \left. \frac{\partial P}{\partial \rho} \right|_s d\rho + \left. \frac{\partial P}{\partial s} \right|_\rho ds \quad (4.6)$$

In the derivations for the speed of sound it was assumed that the flow is isentropic, therefore it can be written

$$\frac{dP}{d\rho} = \left. \frac{\partial P}{\partial \rho} \right|_s \quad (4.7)$$

Note that the equation (4.5) can be obtained by utilizing the momentum equation instead of the energy equation.

Example 4.1:

*Demonstrate that equation (4.5) can be derived from the momentum equation.*

SOLUTION

The momentum equation written for the control volume shown in Figure (4.2) is

$$\overbrace{(P + dP) - P}^{\Sigma F} = \overbrace{(\rho + d\rho)(c - dU)^2 - \rho c^2}^{\int_{cs} U(\rho U dA)} \quad (4.8)$$

Neglecting all the relative small terms results in

$$dP = (\rho + d\rho) \left( c^2 - \cancel{2cdU} + \cancel{dU^2} \right) - \rho c^2 \quad (4.9)$$

$$dP = c^2 d\rho \quad (4.10)$$

This yields the same equation as (4.5).

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End Solution

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### 4.3 Speed of sound in ideal and perfect gases

The speed of sound can be obtained easily for the equation of state for an ideal gas (also perfect gas as a sub set) because of a simple mathematical expression. The pressure for an ideal gas can be expressed as a simple function of density,  $\rho$ , and a function "molecular structure" or ratio of specific heats,  $k$  namely

$$P = \text{constant} \times \rho^k \quad (4.11)$$

and hence

$$c = \sqrt{\frac{dP}{d\rho}} = k \times \text{constant} \times \rho^{k-1} = k \times \frac{\overbrace{\text{constant} \times \rho^k}^P}{\rho} = k \times \frac{P}{\rho} \quad (4.12)$$

Remember that  $P/\rho$  is defined for an ideal gas as  $RT$ , and equation (4.12) can be written as

$$c = \sqrt{kRT} \quad (4.13)$$

Example 4.2:

Calculate the speed of sound in water vapor at 20[bar] and 350°C, (a) utilizes the steam table (b) assuming ideal gas.

SOLUTION

The solution can be estimated by using the data from steam table<sup>3</sup>

$$c \sim \sqrt{\frac{\Delta P}{\Delta \rho}_{s=\text{constant}}} \quad (4.14)$$

$$\text{At } 20[\text{bar}] \text{ and } 350^\circ\text{C}: s = 6.9563 \left[ \frac{\text{kJ}}{\text{K kg}} \right] \quad \rho = 6.61376 \left[ \frac{\text{kg}}{\text{m}^3} \right]$$

$$\text{At } 18[\text{bar}] \text{ and } 350^\circ\text{C}: s = 7.0100 \left[ \frac{\text{kJ}}{\text{K kg}} \right] \quad \rho = 6.46956 \left[ \frac{\text{kg}}{\text{m}^3} \right]$$

$$\text{At } 18[\text{bar}] \text{ and } 300^\circ\text{C}: s = 6.8226 \left[ \frac{\text{kJ}}{\text{K kg}} \right] \quad \rho = 7.13216 \left[ \frac{\text{kg}}{\text{m}^3} \right]$$

After interpretation of the temperature:

$$\text{At } 18[\text{bar}] \text{ and } 335.7^\circ\text{C}: s \sim 6.9563 \left[ \frac{\text{kJ}}{\text{K kg}} \right] \quad \rho \sim 6.94199 \left[ \frac{\text{kg}}{\text{m}^3} \right]$$

and substituting into the equation yields

$$c = \sqrt{\frac{200000}{0.32823}} = 780.5 \left[ \frac{\text{m}}{\text{sec}} \right] \quad (4.15)$$

for ideal gas assumption (data taken from Van Wylen and Sontag, Classical Thermodynamics, table A 8.)

$$c = \sqrt{kRT} \sim \sqrt{1.327 \times 461 \times (350 + 273)} \sim 771.5 \left[ \frac{\text{m}}{\text{sec}} \right]$$

Note that a better approximation can be done with a steam table, and it

End Solution

<sup>3</sup>This data is taken from Van Wylen and Sontag "Fundamentals of Classical Thermodynamics" 2nd edition

Example 4.3:

The temperature in the atmosphere can be assumed to be a linear function of the height for some distances. What is the time it take for sound to travel from point "A" to point "B" under this assumption.?

SOLUTION

The temperature is denoted at "A" as  $T_A$  and temperature in "B" is  $T_B$ . The distance between "A" and "B" is denoted as  $h$ .

$$T(x) = T_A + \frac{x}{h} (T_B - T_A) = T_A + \frac{x}{h} \left( \frac{T_B}{T_A} - 1 \right) T_A \quad (4.16)$$

Where  $x$  is the variable distance. It can be noticed<sup>4</sup> that the controlling dimension is the ratio of the edge temperatures. It can be further noticed that the square root of this ratio is affecting parameter and thus this ratio can be defined as

$$\omega = \sqrt{\frac{T_B}{T_A}} \quad (4.17)$$

Using the definition (4.17) in equation (4.16) results in

$$T(x) = T_A \left( 1 + \frac{\omega^2 - 1}{h} x \right) \quad (4.18)$$

It should be noted that velocity is provided as a function of the distance and not the time (another reverse problem). For an infinitesimal time  $d\tau$  is equal to

$$d\tau = \frac{dx}{\sqrt{kRT(x)}} = \frac{dx}{\sqrt{kRT_A \left( 1 + \frac{\omega^2 - 1}{h} x \right)}}$$

or the integration the about equation as

$$\int_0^t d\tau = \int_0^h \frac{dx}{\sqrt{kRT_A \left( 1 + \frac{\omega^2 - 1}{h} x \right)}}$$

The result of the integration of the above equation yields

$$t_{corrected} = \frac{2h}{(\omega + 1) \sqrt{kRT_A}} \quad (4.19)$$

<sup>4</sup>This suggestion was proposed by Heru Reksoprodjo from Helsinki University of Technology, Finland.

For assumption of constant temperature the time is

$$t = \frac{h}{\sqrt{kRT_A}} \quad (4.20)$$

Hence the correction factor

$$\frac{t_{corrected}}{t} = \frac{2}{(w+1)} \quad (4.21)$$

This correction factor approaches one when  $T_B \rightarrow T_A$  because  $\omega \rightarrow 1$ .

Another possible question<sup>5</sup> to find the temperature,  $T_C$ , where The “standard” equation can be used.

$$\frac{h}{\sqrt{kRT_C}} = \frac{2h}{(w+1)\sqrt{kRT_A}}$$

The above equation leads to

$$T_C = \frac{T_A + T_B + 2\sqrt{T_A T_B}}{4}$$

The explanation to the last equation is left as exercise to the reader.

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End Solution

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#### 4.4 Speed of Sound in Real Gas

The ideal gas model can be improved by introducing the compressibility factor. The compressibility factor represents the deviation from the ideal gas.

Thus, a real gas equation can be expressed in many cases as

$$P = z\rho RT \quad (4.22)$$

The speed of sound of any gas is provided by equation (4.7). To obtain the expression for a gas that obeys the law expressed by (4.22) some mathematical expressions are needed. Recalling from thermodynamics, the Gibbs function (4.23) is used to obtain

$$Tds = dh - \frac{dP}{\rho} \quad (4.23)$$

The definition of pressure specific heat for a pure substance is

$$C_p = \left( \frac{\partial h}{\partial T} \right)_P = T \left( \frac{\partial s}{\partial T} \right)_P \quad (4.24)$$

The definition of volumetric specific heat for a pure substance is

$$C_v = \left( \frac{\partial u}{\partial T} \right)_\rho = T \left( \frac{\partial s}{\partial T} \right)_\rho \quad (4.25)$$

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<sup>5</sup>Indirectly was suggested by Heru Reksoprodjo.

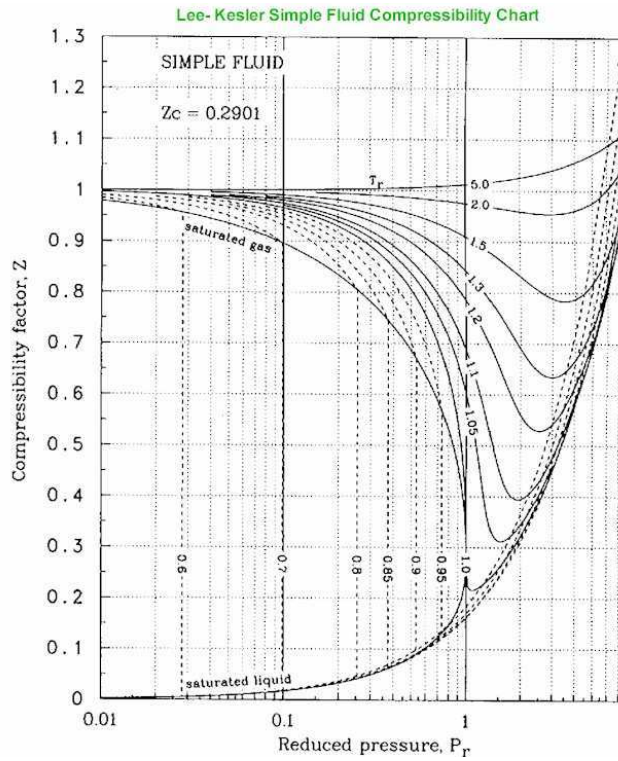


Fig. -4.3. The Compressibility Chart

From thermodynamics, it can be shown <sup>6</sup>

$$dh = C_p dT + \left[ v - T \left( \frac{\partial v}{\partial T} \right)_P \right] dP \quad (4.26)$$

The specific volumetric is the inverse of the density as  $v = zRT/P$  and thus

$$\left( \frac{\partial v}{\partial T} \right)_P = \left( \frac{\partial \left( \frac{zRT}{P} \right)}{\partial T} \right)_P = \frac{RT}{P} \left( \frac{\partial z}{\partial T} \right)_P + \frac{zR}{P} \left( \frac{\partial T}{\partial T} \right)_P \quad (4.27)$$

Substituting the equation (4.27) into equation (4.26) results

$$dh = C_p dT + \left[ v - T \left( \frac{\frac{z}{z} RT}{P} \left( \frac{\partial z}{\partial T} \right)_P + \frac{\frac{v}{T} zR}{P} \right) \right] dP \quad (4.28)$$

<sup>6</sup>See Van Wylen p. 372 SI version, perhaps to insert the discussion here.

Simplifying equation (4.28) to become

$$dh = C_p dT - \left[ \frac{Tv}{z} \left( \frac{\partial z}{\partial T} \right)_P \right] dP = C_p dT - \frac{T}{z} \left( \frac{\partial z}{\partial T} \right)_P \frac{dP}{\rho} \quad (4.29)$$

Utilizing Gibbs equation (4.23)

$$\begin{aligned} Tds &= \overbrace{C_p dT - \frac{T}{z} \left( \frac{\partial z}{\partial T} \right)_P \frac{dP}{\rho}}^{dh} - \frac{dP}{\rho} = C_p dT - \frac{dP}{\rho} \left[ \frac{T}{z} \left( \frac{\partial z}{\partial T} \right)_P + 1 \right] \\ &= C_p dT - \frac{dP}{P} \overbrace{\frac{P}{\rho}}^{zRT} \left[ \frac{T}{z} \left( \frac{\partial z}{\partial T} \right)_P + 1 \right] \end{aligned} \quad (4.30)$$

Letting  $ds = 0$  for isentropic process results in

$$\frac{dT}{T} = \frac{dP}{P} \frac{R}{C_p} \left[ z + T \left( \frac{\partial z}{\partial T} \right)_P \right] \quad (4.31)$$

Equation (4.31) can be integrated by parts. However, it is more convenient to express  $dT/T$  in terms of  $C_v$  and  $d\rho/\rho$  as follows

$$\frac{dT}{T} = \frac{d\rho}{\rho} \frac{R}{C_v} \left[ z + T \left( \frac{\partial z}{\partial T} \right)_\rho \right] \quad (4.32)$$

Equating the right hand side of equations (4.31) and (4.32) results in

$$\frac{d\rho}{\rho} \frac{R}{C_v} \left[ z + T \left( \frac{\partial z}{\partial T} \right)_\rho \right] = \frac{dP}{P} \frac{R}{C_p} \left[ z + T \left( \frac{\partial z}{\partial T} \right)_P \right] \quad (4.33)$$

Rearranging equation (4.33) yields

$$\frac{d\rho}{\rho} = \frac{dP}{P} \frac{C_v}{C_p} \left[ \frac{z + T \left( \frac{\partial z}{\partial T} \right)_P}{z + T \left( \frac{\partial z}{\partial T} \right)_\rho} \right] \quad (4.34)$$

If the terms in the braces are constant in the range under interest in this study, equation (4.34) can be integrated. For short hand writing convenience,  $n$  is defined as

$$n = \frac{\overbrace{C_p}^k}{C_v} \left( \frac{z + T \left( \frac{\partial z}{\partial T} \right)_\rho}{z + T \left( \frac{\partial z}{\partial T} \right)_P} \right) \quad (4.35)$$

Note that  $n$  approaches  $k$  when  $z \rightarrow 1$  and when  $z$  is constant. The integration of equation (4.34) yields

$$\left( \frac{\rho_1}{\rho_2} \right)^n = \frac{P_1}{P_2} \quad (4.36)$$

Equation (4.36) is similar to equation (4.11). What is different in these derivations is that a relationship between coefficient  $n$  and  $k$  was established. This relationship (4.36) isn't new, and in-fact any thermodynamics book shows this relationship. But the definition of  $n$  in equation (4.35) provides a tool to estimate  $n$ . Now, the speed of sound for a real gas can be obtained in the same manner as for an ideal gas.

$$\frac{dP}{d\rho} = nzRT \quad (4.37)$$

Example 4.4:

Calculate the speed of sound of air at 30°C and atmospheric pressure  $\sim 1[\text{bar}]$ . The specific heat for air is  $k = 1.407$ ,  $n = 1.403$ , and  $z = 0.995$ .

Make the calculation based on the ideal gas model and compare these calculations to real gas model (compressibility factor). Assume that  $R = 287[\text{j/kg/K}]$ .

SOLUTION

According to the ideal gas model the speed of sound should be

$$c = \sqrt{kRT} = \sqrt{1.407 \times 287 \times 300} \sim 348.1[\text{m/sec}]$$

For the real gas first coefficient  $n = 1.403$  has

$$c = \sqrt{znRT} = \sqrt{1.403 \times 0.995 \times 287 \times 300} = 346.7[\text{m/sec}]$$

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End Solution

The correction factor for air under normal conditions (atmospheric conditions or even increased pressure) is minimal on the speed of sound. However, a change in temperature can have a dramatical change in the speed of sound. For example, at relative moderate pressure but low temperature common in atmosphere, the compressibility factor,  $z = 0.3$  and  $n \sim 1$  which means that speed of sound is only  $\sqrt{\frac{0.3}{1.4}}$  about factor of (0.5) to calculated by ideal gas model.

## 4.5 Speed of Sound in Almost Incompressible Liquid

Even liquid *normally* is assumed to be incompressible in reality has a small and important compressible aspect. The ratio of the change in the fractional volume to pressure or compression is referred to as the bulk modulus of the material. For example, the average bulk modulus for water is  $2.2 \times 10^9 \text{ N/m}^2$ . At a depth of about 4,000 meters, the pressure is about  $4 \times 10^7 \text{ N/m}^2$ . The fractional volume change is only about 1.8% even under this pressure nevertheless it is a change.

The compressibility of the substance is the reciprocal of the bulk modulus. The amount of compression of almost all liquids is seen to be very small as given in Table (4.5). The mathematical definition of bulk modulus as following

$$B = \rho \frac{dP}{d\rho} \quad (4.38)$$

In physical terms can be written as

$$c = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} = \sqrt{\frac{B}{\rho}} \quad (4.39)$$

For example for water

$$c = \sqrt{\frac{2.2 \times 10^9 \text{ N/m}^2}{1000 \text{ kg/m}^3}} = 1493 \text{ m/s}$$

This agrees well with the measured speed of sound in water, 1482 m/s at 20°C. Many researchers have looked at this velocity, and for purposes of comparison it is given in Table (4.5)

Remark	reference	Value [m/sec]
Fresh Water (20 °C)	Cutnell, John D. & Kenneth W. Johnson. Physics. New York: Wiley, 1997: 468.	1492
Distilled Water at (25 °C)	The World Book Encyclopedia. Chicago: World Book, 1999. 601	1496
Water distilled	Handbook of Chemistry and Physics. Ohio: Chemical Rubber Co., 1967-1968:E37	1494

Table -4.1. Water speed of sound from different sources

The effect of impurity and temperature is relatively large, as can be observed from the equation (4.40). For example, with an increase of 34 degrees from 0°C there is an increase in the velocity from about 1430 m/sec to about 1546 [m/sec]. According to Wilson<sup>7</sup>, the speed of sound in sea water depends on temperature, salinity, and hydrostatic pressure.

Wilson's empirical formula appears as follows:

$$c(S, T, P) = c_0 + c_T + c_S + c_P + c_{STP}, \quad (4.40)$$

<sup>7</sup> J. Acoust. Soc. Amer., 1960, vol.32, N 10, p. 1357. Wilson's formula is accepted by the National Oceanographic Data Center (NODC) USA for computer processing of hydrological information.

where  $c_0 = 1449.14[m/sec]$  is about clean/pure water,  $c_T$  is a function temperature, and  $c_S$  is a function salinity,  $c_P$  is a function pressure, and  $c_{STP}$  is a correction factor between coupling of the different parameters.

material	reference	Value [m/sec]
Glycerol		1904
Sea water	25°C	1533
Mercury		1450
Kerosene		1324
Methyl alcohol		1143
Carbon tetrachloride		926

Table -4.2. Liquids speed of sound, after Aldred, John, Manual of Sound Recording, London: Fountain Press, 1972

#### Example 4.5:

Water in deep sea undergoes compression due to hydrostatic pressure. That is the density is function of the depth. For constant bulk modulus, it was shown in "Basic of Fluid Mechanics" by this author that the density as function of the depth can be estimated as

$$\rho = \frac{\rho_0 B_T}{B_T - g \rho_0 x} \quad (4.V.a)$$

Calculate the time it take for a sound wave to propogate from the surface to a depth  $D$  perpendicular the surface. Assume that no variation of the temperatuere. For the purpose of this excerss, the salinity can be complity ignored.

#### SOLUTION

The assumption that for this excerss that sound velocity is a simple function of density. The speed of sound at any point  $x$  is

$$c = \sqrt{\frac{B_T}{\rho_0 B_T} \frac{B_T}{B_T - g \rho_0 x}} = \sqrt{\frac{B_T - g \rho_0 x}{\rho_0}} \quad (4.V.b)$$

The time takes for the sound the travel the whole distance is the integration of infinitesimal time

$$d\tau = \int_0^t \frac{dx}{\sqrt{\frac{B_T - g \rho_0 x}{\rho_0}}} \quad (4.V.c)$$

The solution of equation (4.V.c) is

$$t = \sqrt{\rho_0} \left( 2\sqrt{B_T} - 2\sqrt{B_T - g \rho_0 x} \right) \quad (4.V.d)$$

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End Solution

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In summary, the speed of sound in liquids is about 3 to 5 relative to the speed of sound in gases.

### 4.6 Speed of Sound in Solids

The situation with solids is considerably more complicated, with different speeds in different directions, in different kinds of geometries, and differences between transverse and longitudinal waves. Nevertheless, the speed of sound in solids is larger than in liquids and definitely larger than in gases.

Young's Modulus for a representative value for the bulk modulus for steel is  $160 \times 10^9 \text{ N/m}^2$ .

material	reference	Value [m/sec]
Diamond		12000
Pyrex glass		5640
Steel	longitudinal wave	5790
Steel	transverse shear	3100
Steel	longitudinal wave (extensional wave)	5000
Iron		5130
Aluminum		5100
Brass		4700
Copper		3560
Gold		3240
Lucite		2680
Lead		1322
Rubber		1600

Table -4.3. Solids speed of sound, after Aldred, John, *Manual of Sound Recording*, London:Fountain Press, 1972

Speed of sound in solid of steel, using a general tabulated value for the bulk modulus, gives a sound speed for structural steel of

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{160 \times 10^9 \text{ N/m}^2}{7860 \text{ Kg/m}^3}} = 4512 \text{ m/s} \quad (4.41)$$

Compared to one tabulated value the example values for stainless steel lays between the speed for longitudinal and transverse waves.

## 4.7 Sound Speed in Two Phase Medium

The gas flow in many industrial situations contains other particles. In actuality, there could be more than one speed of sound for two phase flow. Indeed there is double choking phenomenon in two phase flow. However, for homogeneous and under certain condition a single velocity can be considered. There can be several models that approached this problem. For simplicity, it assumed that two materials are homogeneously mixed. Topic for none homogeneous mixing are beyond the scope of this book. It further assumed that no heat and mass transfer occurs between the particles. In that case, three extreme cases suggest themselves: the flow is mostly gas with drops of the other phase (liquid or solid), about equal parts of gas and the liquid phase, and liquid with some bubbles. The first case is analyzed.

The equation of state for the gas can be written as

$$P_a = \rho_a R T_a \quad (4.42)$$

The average density can be expressed as

$$\frac{1}{\rho_m} = \frac{\xi}{\rho_a} + \frac{1-\xi}{\rho_b} \quad (4.43)$$

where  $\xi = \frac{\dot{m}_b}{\dot{m}}$  is the mass ratio of the materials.

For small value of  $\xi$  equation (4.43) can be approximated as

$$\frac{\rho}{\rho_a} = 1 + m \quad (4.44)$$

where  $m = \frac{\dot{m}_b}{\dot{m}_a}$  is mass flow rate per gas flow rate.

The gas density can be replaced by equation (4.42) and substituted into equation (4.44)

$$\frac{P}{\rho} = \frac{R}{1+m} T \quad (4.45)$$

A approximation of addition droplets of liquid or dust (solid) results in reduction of  $R$  and yet approximate equation similar to ideal gas was obtained. It must noticed that  $m = \text{constant}$ . If the droplets (or the solid particles) can be assumed to have the same velocity as the gas with no heat transfer or friction between the particles isentropic relation can be assumed as

$$\frac{P}{\rho_a^k} = \text{constant} \quad (4.46)$$

Assuming that partial pressure of the particles is constant and applying the second law for the mixture yields

$$0 = \overbrace{mC}^{\text{droplets}} \frac{dT}{T} + \overbrace{C_p}^{\text{gas}} \frac{dT}{T} - R \frac{dP}{P} = \frac{(C_p + mC)dT}{T} - R \frac{dP}{P} \quad (4.47)$$

Therefore, the mixture isentropic relationship can be expressed as

$$\frac{P^{\frac{\gamma-1}{\gamma}}}{T} = \text{constant} \quad (4.48)$$

where

$$\frac{\gamma-1}{\gamma} = \frac{R}{C_p + mC} \quad (4.49)$$

Recalling that  $R = C_p - C_v$  reduces equation (4.49) into

$$\gamma = \frac{C_p + mC}{C_v + mC} \quad (4.50)$$

In a way the definition of  $\gamma$  was so chosen that effective specific pressure heat and effective specific volumetric heat are  $\frac{C_p+mC}{1+m}$  and  $\frac{C_v+mC}{1+m}$  respectively. The correction factors for the specific heat is not linear.

Since the equations are the same as before hence the familiar equation for speed of sound can be applied as

$$c = \sqrt{\gamma R_{mix} T} \quad (4.51)$$

It can be noticed that  $R_{mix}$  and  $\gamma$  are smaller than similar variables in a pure gas. Hence, this analysis results in lower speed of sound compared to pure gas. Generally, the velocity of mixtures with large gas component is smaller of the pure gas. For example, the velocity of sound in slightly wet steam can be about one third of the pure steam speed of sound.

## Meta

For a mixture of two phases, speed of sound can be expressed as

$$c^2 = \frac{\partial P}{\partial \rho} = \frac{\partial P[f(X)]}{\partial \rho} \quad (4.52)$$

where  $X$  is defined as

$$X = \frac{s - s_f(P_B)}{s_{fg}(P_B)} \quad (4.53)$$

## Meta End